RELATIVE MOTION

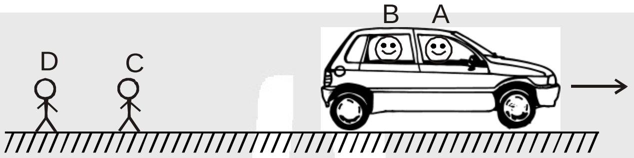
1. RELATIVE MOTION

Motion is a combined property of the object under study as well as the observer. It is always relative ; there is no such thing like absolute motion or absolute rest. Motion is always defined with respect to an observer or reference frame.

Reference frame :

Reference frame is an axis system from which motion is observed along with a clock attached to the axis, to measure time. Reference frame can be stationary or moving.

☞ Suppose there are two persons A and B sitting in a car moving at constant speed. Two stationary persons C and D observe them from the ground.



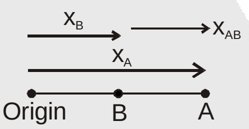
Here B appears to be moving for C and D, but at rest for A. Similarly C appears to be at rest for D but moving backward for A and B.

2. RELATIVE MOTION IN ONE DIMENSION :

2.1. Relative Position :

It is the position of a particle w.r.t. observer.

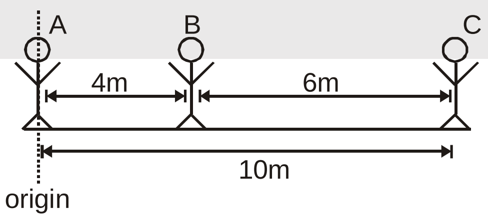
In general if position of A w.r.t. to origin is x_A and that of B w.r.t. origin is x_B then "Position of A w.r.t. B" x_{AB} is



$$x_{AB} = x_A - x_B$$

Solved Example

Example 1. See the figure (take +ve direction towards right and –ve towards left). Find x_{BA}, x_{CA}, x_{CB}, x_{AB} and x_{AC}.



Here, Position of B w.r.t. A is 4 m towards right. (x_{BA} = +4m)

Position of C w.r.t. A is 10 m towards right. (x_{CA} = +10m)

Position of C w.r.t. B is 6 m towards right (x_{CB} = +6m)

Position of A w.r.t. B is 4 m towards left. (x_{AB} = –4 m)

Position of A w.r.t. C is 10 m towards left. (x_{AC} = –10m)

2.2. Relative Velocity

Definition : Relative velocity of a particle A with respect to B is defined as the velocity with which A appears to move if B is considered to be at rest. In other words, it is the velocity with which A appears to move as seen by B considering itself to be at rest.

NOTE 1 : All velocities are relative & have no significance unless observer is specified. However, when we say "velocity of A", what we mean is, velocity of A w.r.t. ground which is assumed to be at rest.

Relative velocity in one dimension -

If x_A is the position of A w.r.t. ground, x_B is position of B w.r.t. ground and x_{AB} is position of A w.r.t. B then we can say v_A = velocity of A w.r.t. ground = $\frac{dx_A}{dt}$

v_B = velocity of B w.r.t. ground = $\frac{dx_B}{dt}$

and v_{AB} = velocity of A w.r.t. B = $\frac{dx_{AB}}{dt} = \frac{d}{dt}(x_A - x_B) = \frac{dx_A}{dt} - \frac{dx_B}{dt}$

Thus $\boxed{v_{AB} = v_A - v_B}$

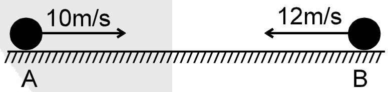
NOTE 2 : Velocity of an object w.r.t. itself is always zero.

Solved Example

Example 2. An object A is moving with 5 m/s and B is moving with 20 m/s in the same direction. (Positive x-axis)
(i) Find velocity of B with respect to A.
(ii) Find velocity of A with respect to B

Solution :
(i) $v_B = +20$ m/s, $v_A = +5$ m/s,
$v_{BA} = v_B - v_A = +15$ m/s
(ii) $v_B = +20$ m/s, $v_A = +15$ m/s
$v_{AB} = v_A - v_B = -15$ m/s
Note : $v_{BA} = -v_{AB}$

Example 3. Two objects A and B are moving towards each other with velocities 10 m/s and 12 m/s respectively as shown.



(i) Find the velocity of A with respect to B.
(ii) Find the velocity of B with respect to A

Solution : $v_A = +10$, $v_B = -12$
(i) $v_{AB} = v_A - v_B = (10) - (-12) = 22$ m/s.
(ii) $v_{BA} = v_B - v_A = (-12) - (10) = -22$ m/s.

2.3. Relative Acceleration

It is the rate at which relative velocity is changing.

$$a_{AB} = \frac{dv_{AB}}{dt} = \frac{dv_A}{dt} - \frac{dv_B}{dt} = a_A - a_B$$

Equations of motion when relative acceleration is constant.

$v_{rel} = u_{rel} + a_{rel}\, t$

$s_{rel} = u_{rel}\, t + \frac{1}{2} a_{rel}\, t^2$

$v^2_{rel} = u^2_{rel} + 2a_{rel}\, s_{rel}$

2.4. Velocity of Approach / Separation

It is the component of relative velocity of one particle w.r.t. another, along the line joining them. If the separation is decreasing, we say it is velocity of approach and if separation is increasing, then we say it is velocity of separation. In one dimension, since relative velocity is along the line joining A and B, hence velocity of approach / separation is simply equal to magnitude of relative velocity of A w.r.t. B.

Solved Example

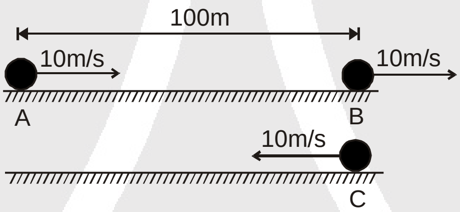
Example 4. A particle A is moving with a speed of 10 m/s towards right and another particle B is moving at speed of 12 m/s towards left. Find their velocity of approach.

Solution : $V_A = +10$, $V_B = -12 \Rightarrow V_{AB} = V_A - V_B \Rightarrow 10 - (-12) = 22$ m/s
since separation is decreasing hence $V_{app} = |V_{AB}| = 22$ m/s
Ans. 22 m/s

Example 5. A particle A is moving with a speed of 20 m/s towards right and another particle B is moving at a speed of 5 m/s towards right. Find their velocity of approach.

Solution : $V_A = +20$, $V_B = +5$
$V_{AB} = V_A - V_B$
$20 - (+5) = 15$ m/s
since separation is decreasing hence $V_{app} = |V_{AB}| = 15$ m/s
Ans. 15 m/s

Example 6. A particle A is moving with a speed of 10 m/s towards right, particle B is moving at a speed of 10 m/s towards right and another particle C is moving at speed of 10 m/s towards left. The separation between A and B is 100 m. Find the time interval between C meeting B and C meeting A.



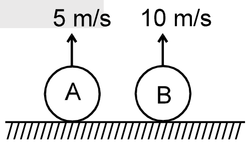
Solution : $t = \dfrac{\text{seperation between A and C}}{V_{app} \text{ of A and C}} = \dfrac{100}{10-(-10)} = 5$ sec.

Ans. : 5 sec.

Note : $a_{app} = \left(\dfrac{d}{dt}\right)v_{app}$, $a_{sep} = \dfrac{d}{dt}v_{sep}$

$v_{app} = \int a_{app}dt$, $v_{sep} = \int a_{sep}dt$

Example 7. A and B are thrown vertically upward with velocity, 5 m/s and 10 m/s respectively (g = 10 m/s^2. Find separation between them after one second



Solution : $S_A = ut - \frac{1}{2}gt^2$

$= 5t - \frac{1}{2} \times 10 \times t^2 = 5 \times 1 - 5 \times 1^2 = 5 - 5 = 0$

$S_B = ut - \frac{1}{2}gt^2 = 10 \times 1 - \frac{1}{2} \times 10 \times 1^2 = 10 - 5 = 5$

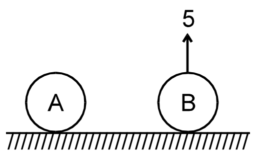
$\therefore S_B - S_A$ = separation = 5m.

Aliter : $\vec{a}_{BA} = \vec{a}_B - \vec{a}_A = (-10) - (-10) = 0$

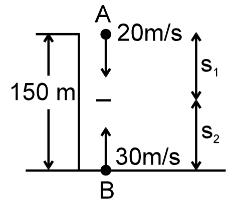
Also $\vec{v}_{BA} = \vec{v}_B - \vec{v}_A = 10 - 5 = 5$ m/s

$\therefore \vec{s}_{BA}$ (in 1 sec) $= \vec{v}_{BA} \times t = 5 \times 1 = 5$ m

$\therefore$ Distance between A and B after 1 sec = 5 m.



Example 8. A ball is thrown downwards with a speed of 20 m/s from the top of a building 150 m high and simultaneously another ball is thrown vertically upwards with a speed of 30 m/s from the foot of the building. Find the time after which both the balls will meet. ($g = 10\ m/s^2$)



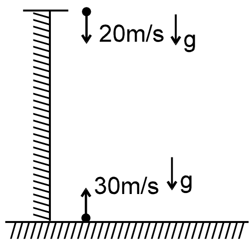
Solution : $S_1 = 20\,t + 5\,t^2$

$S_2 = 30\,t - 5\,t^2$

$S_1 + S_2 = 150$

$\Rightarrow\ 150 = 50\,t \ \Rightarrow\ t = 3\ s$

Aliter : Relative acceleration of both is zero since both have same acceleration in downward direction



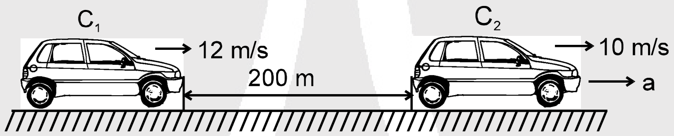
$\vec{a}_{AB} = \vec{a}_A - \vec{a}_B = g - g = 0$

$\vec{v}_{BA} = 30 - (-20) = 50$

$s_{BA} = v_{BA} \times t$

$t = \dfrac{s_{BA}}{v_{BA}} = \dfrac{150}{50} = 3\ s$

Example 9. Two cars C_1 and C_2 moving in the same direction on a straight single lane road with velocities 12 m/s and 10 m/s respectively. When the separation between the two was 200 m C_2 started accelerating to avoid collision. What is the minimum acceleration of car C_2 so that they don't collide.



Solution : Acceleration of car 1 w.r.t. car 2

$\vec{a}_{12} = \vec{a}_1 - \vec{a}_2 = \vec{a}_{C_1} - \vec{a}_{C_2} = 0 - a = (-a)$

$\vec{u}_{12} = \vec{u}_1 - \vec{u}_2 = 12 - 10 = 2\ m/s.$

The collision is just avoided if relative velocity becomes zero just at the moment the two cars meet each other.

i.e. $v_{12} = 0$ When $s_{12} = 200$

Now $v_{12} = 0$, $\vec{u}_{12} = 2$, $\vec{a}_{12} = -a$ and $s_{12} = 200$

$\therefore\ v_{12}^2 - u_{12}^2 = 2a_{12}s_{12}$

$\Rightarrow\ 0 - 2^2 = -2 \times a \times 200 \qquad \Rightarrow \qquad a = \dfrac{1}{100}\ m/s^2 = 0.1\ m/s^2 = 1\ cm/s^2.$

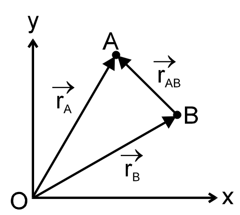
$\therefore$ Minimum acceleration needed by car $C_2 = 1\ cm/s^2$

3. RELATIVE MOTION IN TWO DIMENSION

$\vec{r}_A$ = position of A with respect to O

$\vec{r}_B$ = position of B with respect to O

$\vec{r}_{AB}$ = position of A with respect to B.



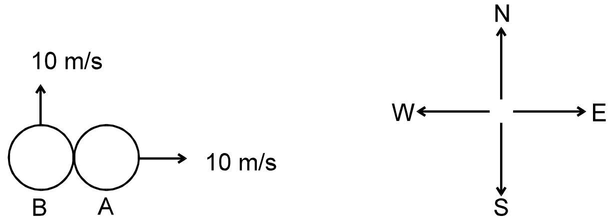
$\vec{r}_{AB} = \vec{r}_A - \vec{r}_B$ (The vector sum $\vec{r}_A - \vec{r}_B$ can be done by Δ law of addition or resolution method)

$\therefore\ \dfrac{d(\vec{r}_{AB})}{dt} = \dfrac{d(\vec{r}_A)}{dt} - \dfrac{d(\vec{r}_B)}{dt}.$

$\Rightarrow\ \vec{v}_{AB} = \vec{v}_A - \vec{v}_B\ ;\ \dfrac{d(\vec{v}_{AB})}{dt} = \dfrac{d(\vec{v}_A)}{dt} - \dfrac{d(\vec{v}_B)}{dt} \ \Rightarrow\ \vec{a}_{AB} = \vec{a}_A - \vec{a}_B$

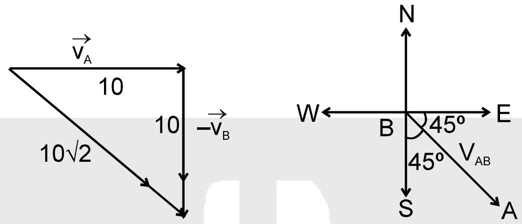
Solved Example

Example 10. Object A and B both have speed of 10 m/s. A is moving towards East while B is moving towards North starting from the same point as shown. Find velocity of A relative to B.



Solution :

Method 1 : $\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$ $\therefore$ $|\vec{v}_{AB}| = 10\sqrt{2}$

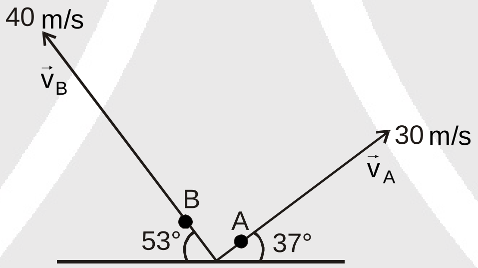


Method 2 : $\vec{v}_A = 10\,\hat{i}$, $\vec{v}_B = 10\,\hat{j}$

$\vec{v}_{AB} = \vec{v}_A - \vec{v}_B = 10\,\hat{i} - 10\,\hat{j}$ $\therefore$ $|\vec{v}_{AB}| = 10\sqrt{2}$

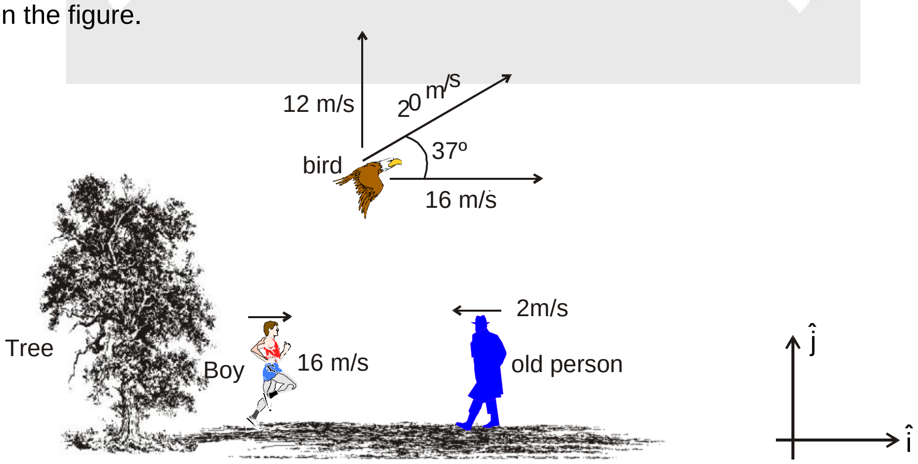
Note : $|\vec{v}_A - \vec{v}_B| = \sqrt{v_A^2 + v_B^2 - 2v_A v_B \cos\theta}$, where θ is angle between $\vec{v}_A$ and $\vec{v}_B$.

Example 11. Two particles A and B are projected in air. A is thrown with a speed of 30 m/sec and B with a speed of 40 m/sec as shown in the figure. What is the separation between them after 1 sec.



Solution : $\vec{a}_{AB} = \vec{a}_A - \vec{a}_B = \vec{g} - \vec{g} = 0$ $\therefore$ $v_{AB} = \sqrt{30^2 + 40^2} = 50$ $\therefore$ $s_{AB} = v_{AB}t = 50\,t = 50$ m

Example 12. An old man and a boy are walking towards each other and a bird is flying over them as shown in the figure.



(1) Find the velocity of tree, bird and old man as seen by boy.
(2) Find the velocity of tree, bird and boy as seen by old man
(3) Find the velocity of tree, boy and old man as seen by bird.

Solution :

(1) With respect to boy :

v_{tree} = 16 m/s (←) or $-16\hat{i}$

v_{bird} = 12 m/s (↑) or $12\hat{j}$

$v_{old\ man}$ = 18 m/s (←) or $-18\hat{i}$

(2) With respect to old man :

v_{Boy} = 18 m/s (→) or $18\hat{i}$

v_{Tree} = 2 m/s (→) or $2\hat{i}$

v_{Bird} = 18 m/s (→) and 12 m/s (↑) or $18\hat{i} + 12\hat{j}$

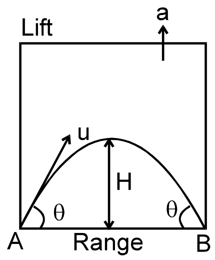
(3) With respect to Bird : v_{Tree} = 12 m/s (↓) and 16 m/s (←) or $-12\hat{j} - 16\hat{i}$

$v_{old\ man}$ = 18 m/s (←) and 12 m/s (↓) or $-18\hat{i} - 12\hat{j}$

v_{Boy} = 12 m/s (↓) or $-12\hat{j}$

3.2. Relative Motion in Lift

Projectile motion in a lift moving with acceleration a upwards



(1) In the reference frame of lift, acceleration of a freely falling object is (g + a)

(2) Velocity at maximum height = ucosθ

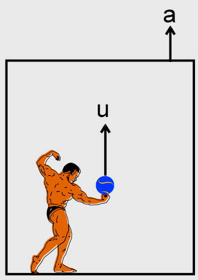
(3) $T = \frac{2u\sin\theta}{g+a}$

(4) Maximum height (H) = $\frac{u^2\sin^2\theta}{2(g+a)}$

(5) Range = $\frac{u^2\sin 2\theta}{g+a}$

Solved Example

Example 13. A lift is moving up with acceleration a. A person inside the lift throws the ball upwards with a velocity u relative to hand.



(a) What is the time of flight of the ball?

(b) What is the maximum height reached by the ball in the lift?

Solution :

(a) $\vec{a}_{BL} = \vec{a}_B - \vec{a}_L = g + a$

$\vec{s} = \vec{u}\,t + \frac{1}{2}\vec{a}_{BL}t^2$

$0 = uT - 1/2\,(g + a)T^2$

$\therefore \quad T = \frac{2u}{(g+a)}$

(b) $v^2 - u^2 = 2as$

$0 - u^2 = -2(g + a)\,H$

$H = \frac{u^2}{2(g+a)}$

4. RELATIVE MOTION IN RIVER FLOW

If a man can swim relative to water with velocity $\vec{v}_{mR}$ and water is flowing relative to ground with velocity $\vec{v}_R$, velocity of man relative to ground $\vec{v}_m$ will be given by :

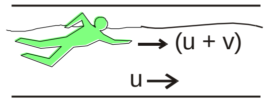
$\vec{v}_{mR} = \vec{v}_m - \vec{v}_R$ or $\vec{v}_m = \vec{v}_{mR} + \vec{v}_R$

If $\vec{v}_R = 0$, then $\vec{v}_m = \vec{v}_{mR}$ in words, velocity of man in still water = velocity of man w.r.t. river

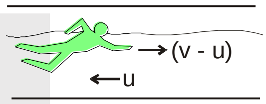
4.1. River Problem in One Dimension :

☞ Velocity of river is u & velocity of man in still water is v.

Case (i) : Man swimming downstream (along the direction of river flow). In this case velocity of river $v_R = +u$
velocity of man w.r.t. river $v_{mR} = +v$
now $\vec{v}_m = \vec{v}_{mR} + \vec{v}_R = u + v$



Case (ii) : Man swimming upstream (opposite to the direction of river flow). In this case velocity of river $\vec{v}_R = -u$
velocity of man w.r.t. river $\vec{v}_{mR} = +v$
now $\vec{v}_m = \vec{v}_{mR} + \vec{v}_R = (v - u)$



Solved Example

Example 14. A swimmer capable of swimming with velocity 'v' relative to water jumps in a flowing river having velocity 'u'. The man swims a distance d down stream and returns back to the original position. Find out the time taken in complete motion.

Solution : Total time = time of swimming downstream + time of swimming upstream

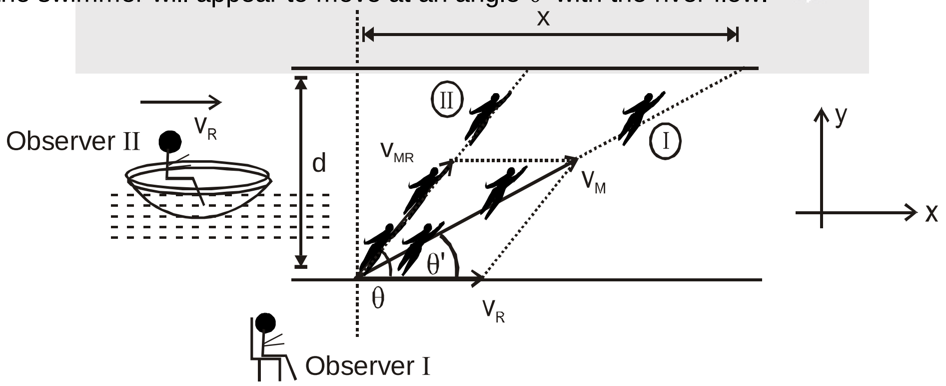
$$t = t_{down} + t_{up} = \frac{d}{v+u} + \frac{d}{v-u} = \frac{2dv}{v^2 - u^2}$$ **Ans.**

4.2. Motion of Man Swimming in a River

Consider a man swimming in a river with a velocity of $\vec{v}_{MR}$ relative to river at an angle of θ with the river flow. The velocity of river is $\vec{V}_R$. Let there be two observers I and II, observer I is on ground and observer II is on a raft floating along with the river and hence moving with the same velocity as that of river. Hence motion w.r.t. observer II is same as motion w.r.t. river. i.e., the man will appear to swim at an angle θ with the river flow for observer II .

For observer I the velocity of swimmer will be $\vec{v}_M = \vec{v}_{MR} + \vec{v}_R$,

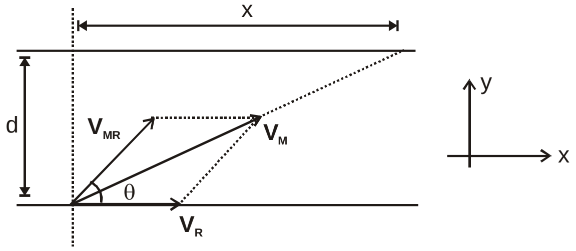
Hence the swimmer will appear to move at an angle θ' with the river flow.



(I) : Motion of swimmer for observer I
(II) : Motion of swimmer for observer II

4.3. River problem in two dimension (crossing river) :

Consider a man swimming in a river with a velocity of $\vec{v}_{MR}$ relative to river at an angle of θ with the river flow. The velocity of river is V_R and the width of the river is d



$\vec{v}_M = \vec{v}_{MR} + \vec{v}_R \Rightarrow \vec{v}_M = (v_{MR}\cos\theta\,\hat{i} + v_{MR}\sin\theta\,\hat{j}) + v_R\,\hat{i} \Rightarrow \vec{v}_M = (v_{MR}\cos\theta + v_R)\,\hat{i} + v_{MR}\sin\theta\,\hat{j}$

Here $v_{MR}\sin\theta$ is the component of velocity of man in the direction perpendicular to the river flow. This component of velocity is responsible for the man crossing the river. Hence if the time to cross the river is t, then $t = \frac{d}{v_y} = \frac{d}{v_{MR}\sin\theta}$

DRIFT

It is defined as the displacement of man in the direction of river flow. (See the figure). It is simply the displacement along x-axis, during the period the man crosses the river. $(v_{MR}\cos\theta + v_R)$ is the component of velocity of man in the direction of river flow and this component of velocity is responsible for drift along the river flow. If drift is x then,

Drift = $v_x \times t$

$$x = (v_{MR}\cos\theta + v_R) \times \frac{d}{v_{MR}\sin\theta}$$

4.4. Crossing the river in shortest time

As we know that $t = \frac{d}{v_{MR}\sin\theta}$. Clearly t will be minimum when θ = 90° i.e. time to cross the river will be minimum if man swims perpendicular to the river flow. Which is equal to $\frac{d}{v_{MR}}$.

4.5. Crossing the river in shortest path, Minimum Drift

The minimum possible drift is zero. In this case the man swims in the direction perpendicular to the river flow as seen from the ground. This path is known as **shortest path**

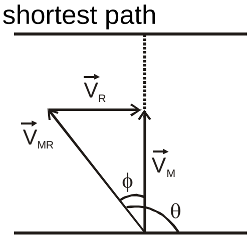
here $x_{min} = 0 \Rightarrow (v_{MR}\cos\theta + v_R) = 0$ or $\cos\theta = -\frac{v_R}{v_{MR}}$

☞ since cosθ is –ve, ∴ θ > 90°, i.e. for minimum drift the man must swim at some angle ϕ with the perpendicular in backward direction. Where $\sin\phi = \frac{v_R}{v_{MR}}$

☞ $\theta = \cos^{-1}\left(\frac{-v_R}{v_{MR}}\right)$ ∴ $\left|\frac{v_R}{v_{MR}}\right| \le 1$ i.e. $v_R \le v_{MR}$

i.e. minimum drift is zero if and only if velocity of man in still water is greater than or equal to the velocity of river.

☞ Time to cross the river along the shortest path



$$t = \frac{d}{v_{MR}\sin\theta} = \frac{d}{\sqrt{v_{MR}^2 - V_R^2}}$$

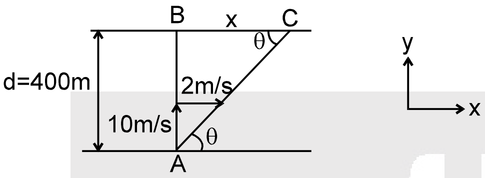
Note :

- **If $v_R > v_{MR}$ then it is not possible to have zero drift. In this case the minimum drift (corresponding to shortest possible path is non zero and the condition for minimum drift can be proved to be $\cos\theta = -\left(\frac{v_{MR}}{v_R}\right)$ or $\sin\phi = \left(\frac{v_{MR}}{v_R}\right)$ for minimum but non zero drift.**

Solved Example

Example 15. A 400 m wide river is flowing at a rate of 2.0 m/s.A boat is sailing with a velocity of 10 m/s with respect to the water, in a direction perpendicular to the river.
(a) Find the time taken by the boat to reach the opposite bank.
(b) How far from the point directly opposite to the starting point does the boat reach the opposite bank.
(c) In what direction does the boat actually move, with river flow (downstream).

Solution :



(a) time taken to cross the river $t = \frac{d}{v_y} = \frac{400m}{10m/s} = 40$ s **Ans.**

(b) drift (x) = $(v_x)(t)$ = (2m/s) (40s) = 80 m **Ans.**

(c) Actual direction of boat, $\theta = \tan^{-1}\left(\frac{10}{2}\right) = \tan^{-1} 5$, (downstream) with the river flow.

Example 16. A man can swim at the rate of 5 km/h in still water. A 1 km wide river flows at the rate of 3 km/h. The man wishes to swim across the river directly opposite to the starting point.
(a) Along what direction must the man swim?
(b) What should be his resultant velocity?
(c) How much time will he take to cross the river?

Solution :

(a) The velocity of man with respect to river v_{mR}= 5 km/hr, this is greater than the river flow velocity, therefore, he can cross the river directly (along the shortest path). The angle of swim must be

$$\theta = \frac{\pi}{2} + \sin^{-1}\left(\frac{v_r}{v_{mR}}\right) = 90^o + \sin^{-1}\left(\frac{v_r}{v_{mR}}\right) = 90^o + \sin^{-1}\left(\frac{3}{5}\right) = 90^o + 37^o$$

= 127º w.r.t. the river flow or 37° w.r.t. perpendicular in backward direction **Ans.**

(b) Resultant velocity will be

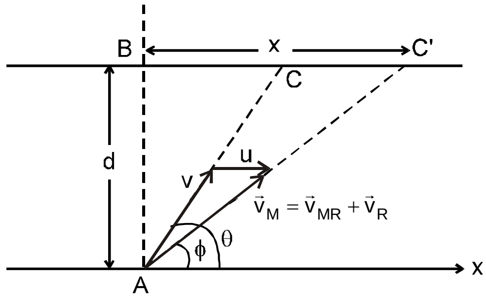
$$v_m = \sqrt{v_{mR}^2 - v_R^2} = \sqrt{5^2 - 3^2} = 4 \text{ km/hr}$$

along the direction perpendicular to the river flow.

(c) time taken to cross the $t = \frac{d}{\sqrt{v_{mR}^2 - v_R^2}} = \frac{1km}{4km/hr} = \frac{1}{4}$ h = 15 min

Example 17. A man wishing to cross a river flowing with velocity u jumps at an angle θ with the river flow.
(i) Find the net velocity of the man with respect to ground if he can swim with speed v in still water.
(ii) In what direction does the man actually move.
(iii) Find how far from the point directly opposite to the starting point does the man reach the opposite bank, if the width of the river is d. (i.e. drift)

Solution :



(i) $v_{MR} = v$, $v_R = u$; $\vec{v}_M = \vec{v}_{MR} + \vec{v}_R$

∴ Velocity of man,

$v_M = \sqrt{u^2 + v^2 + 2vu\cos\theta}$ **Ans.**

(ii) $\tan\phi = \dfrac{v\sin\theta}{u + v\cos\theta}$ **Ans.**

(iii) $(v \sin\theta)\, t = d \Rightarrow t = \dfrac{d}{v\sin\theta}$;

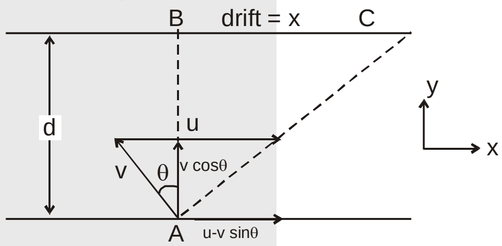
$x = (u + v\cos\theta)\, t = (u + v\cos\theta)\dfrac{d}{v\sin\theta}$

Example 18. A boat moves relative to water with a velocity v which is n times less than the river flow velocity u. At what angle to the stream direction must the boat move to minimize drifting ?

Solution : (In this problem, one thing should be carefully noted that the velocity of boat is less than the river flow velocity. Hence boat cannot reach the point directly opposite to its starting point. i.e., drift can never be zero)

Suppose boat starts at an angle θ from the normal direction up stream as shown.

Component of velocity of boat along the river, $v_x = u - v\sin\theta$



and velocity perpendicular to the river,

$v_y = v\cos\theta$.

time taken to cross the river is

$t = \dfrac{d}{v_y} = \dfrac{d}{v\cos\theta}$.

Drift $x = (v_x)t = (u - v\sin\theta)\dfrac{d}{v\cos\theta}$

$= \dfrac{ud}{v}\sec\theta - d\tan\theta$

The drift x is minimum, when $\dfrac{dx}{d\theta} = 0$,

or $\left(\dfrac{ud}{v}\right)(\sec\theta \,.\, \tan\theta) - d\sec^2\theta = 0$

or $\dfrac{u}{v}\sin\theta = 1$ or $\boxed{\sin\theta = \dfrac{v}{u}}$

This is the result we stated without proof as a note in section 4.5

so, for minimum drift, the boat must move at an angle $\theta = \sin^{-1}\left(\dfrac{v}{u}\right) = \sin^{-1}\dfrac{1}{n}$ from normal direction.

5. WIND AIRPLANE PROBLEMS

This is very similar to boat river flow problems. The only difference is that boat is replaced by aeroplane and river is replaced by wind.

Thus, velocity of aeroplane with respect to wind

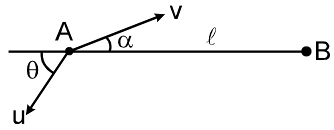
$\vec{v}_{aw} = \vec{v}_a - \vec{v}_w$ or $\vec{v}_a = \vec{v}_{aw} + \vec{v}_w$

where, $\vec{v}_a$ = velocity of aeroplane w.r.t. ground and, $\vec{v}_w$ = velocity of wind.

Solved Example

Example 19. An aeroplane flies along a straight path A to B and returns back again. The distance between A and B is ℓ and the aeroplane maintains the constant speed v w.r.t. wind. There is a steady wind with a speed u at an angle θ with line AB. Determine the expression for the total time of the trip.

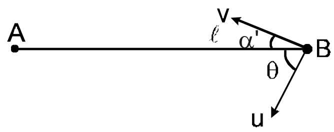
Solution : Suppose plane is oriented at an angle α w.r.t. line AB while the plane is moving from A to B :



Velocity of plane along AB = v cosα – ucosθ,
and for no-drift from line AB ; v sinα = usinθ

$\Rightarrow \quad \sin\alpha = \dfrac{u\sin\theta}{v}$

time taken from A to B : $t_{AB} = \dfrac{\ell}{v\cos\alpha - u\cos\theta}$



Suppose plane is oriented at an angle α' w.r.t. line AB while the plane is moving from B to A :
velocity of plane along BA = vcosα + u cosθ and for no drift from line AB ; vsinα = usinθ

$\Rightarrow \quad \sin\alpha = \dfrac{u\sin\theta}{v} \qquad \Rightarrow \qquad \alpha = \alpha'$

time taken from B to A : $t_{BA} = \dfrac{\ell}{v\cos\alpha + u\cos\theta}$

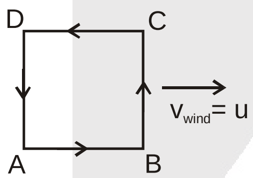
total time taken = $t_{AB} + t_{BA} = \dfrac{\ell}{v\cos\alpha - u\cos\theta} + \dfrac{\ell}{v\cos\alpha + u\cos\theta}$

$$= \frac{2v\ell\cos\alpha}{v^2\cos^2\alpha - u^2\cos^2\theta} = \frac{2v\ell\sqrt{1-\dfrac{u^2\sin^2\theta}{v^2}}}{v^2 - u^2}.$$

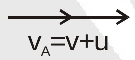
Example 20. Find the time an aeroplane having velocity v, takes to fly around a square with side a if the wind is blowing at a velocity u along one side of the square.

Answer : $\dfrac{2a}{v^2-u^2}\left(v+\sqrt{v^2-u^2}\right)$

Solution :

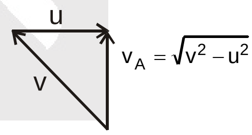


Velocity of aeroplane while flying through AB



$v_A = v + u \quad ; \quad t_{AB} = \dfrac{a}{v+u}$

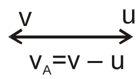
Velocity of aeroplane while flying through BC



$v_A = \sqrt{v^2-u^2}$;

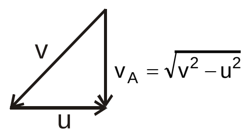
$t_{BC} = \dfrac{a}{\sqrt{v^2-u^2}}$

Velocity of aeroplane while flying through CD



$v_A = v - u \quad ; \quad t_{CD} = \dfrac{a}{v-u}$

Velocity of aeroplane while flying through DA



$v_A = \sqrt{v^2-u^2}$; $t_{DA} = \dfrac{a}{\sqrt{v^2-u^2}}$

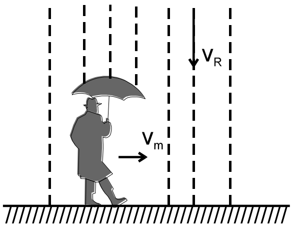
Total time = $t_{AB} + t_{BC} + t_{CD} + t_{DA} = \dfrac{a}{v+u} + \dfrac{a}{\sqrt{v^2-u^2}} + \dfrac{a}{v-u} + \dfrac{a}{\sqrt{v^2-u^2}} = \dfrac{2a}{v^2-u^2}\left(v+\sqrt{v^2-u^2}\right)$

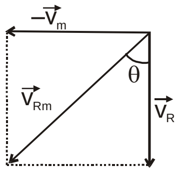
6. RAIN PROBLEM

If rain is falling vertically with a velocity $\vec{v}_R$ and an observer is moving horizontally with velocity $\vec{v}_m$, the velocity of rain relative to observer will be :

$\vec{v}_{Rm} = \vec{v}_R - \vec{v}_m$ or $v_{Rm} = \sqrt{v_R^2 + v_m^2}$

and direction $\theta = \tan^{-1}\left(\frac{v_m}{v_R}\right)$ with the vertical as shown in figure

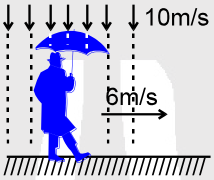




Solved Example

Example 21. Rain is falling vertically at speed of 10 m/s and a man is moving with velocity 6 m/s. Find the angle at which the man should hold his umbrella to avoid getting wet.

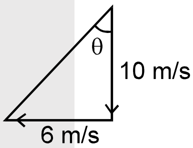
Solution :



$\vec{v}_{rain} = -10\hat{j} \Rightarrow \vec{v}_{man} = 6\hat{j}$

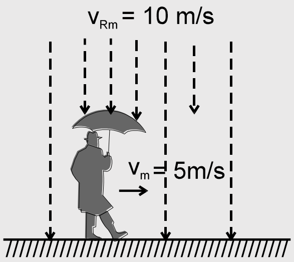
$\vec{v}_{r.w.r.t.\ man} = -10\hat{j} - 6\hat{i}$

$\tan\theta = \frac{6}{10} \Rightarrow \theta = \tan^{-1}\left(\frac{3}{5}\right)$



Where θ is angle with vertical

Example 22. A man moving with 5m/s observes rain falling vertically at the rate of 10 m/s. Find the speed and direction of the rain with respect to ground.



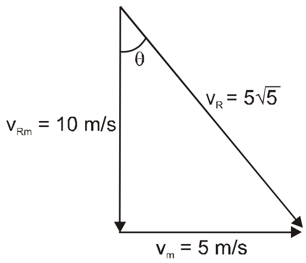
Solution : v_{RM} = 10 m/s, v_M = 5 m/s

$\vec{v}_{RM} = \vec{v}_R - \vec{v}_M$

$\Rightarrow \vec{v}_R = \vec{v}_{RM} + \vec{v}_M$

$\Rightarrow \vec{v}_R = 5\sqrt{5}$

$\tan\theta = \frac{1}{2},\ \theta = \tan^{-1}\frac{1}{2}$.

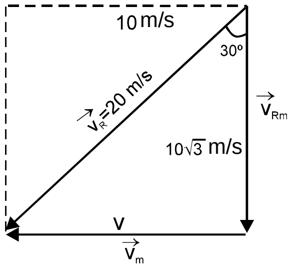


Example 23. A standing man, observes rain falling with velocity of 20 m/s at an angle of 30º with the vertical.

(i) Find the velocity with which the man should move so that rain appears to fall vertically to him.

(ii) Now if he further increases his speed, rain again appears to fall at 30º with the vertical. Find his new velocity.

Solution :



(i) $\vec{v}_m = -v\hat{i}$ (let)

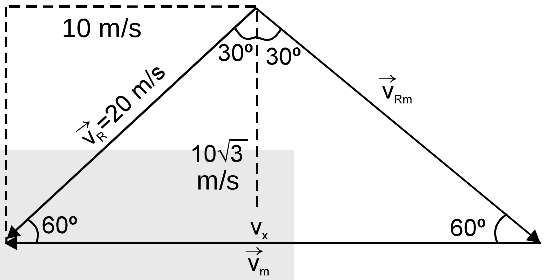
$\vec{v}_R = -10\hat{i} - 10\sqrt{3}\,\hat{j}$

$\vec{v}_{RM} = -(10\hat{i} - v) - 10\sqrt{3}\,\hat{j}$

$\Rightarrow\ -(10 - v) = 0$

(for vertical fall, horizontal component must be zero)

or v = 10 m/s **Ans.**



(ii) $\vec{v}_R = -10\hat{i} - 10\sqrt{3}\,\hat{j}$

$\vec{v}_m = -v_x\hat{i}$

$\vec{v}_{RM} = -(10 - v_x)\hat{i} - 10\sqrt{3}\,\hat{j}$

Angle with the vertical = 30º

$\Rightarrow\ \tan 30º = \dfrac{10 - v_x}{-10\sqrt{3}} \quad \Rightarrow\ v_x = 20$ m/s

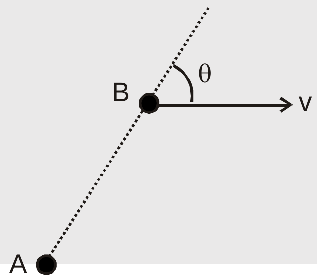
7. VELOCITY OF APPROACH / SEPARATION IN TWO DIMENSION

It is the component of relative velocity of one particle w.r.t. another, along the line joining them.

If the separation is decreasing, we say it is velocity of approach and if separation is increasing, then we say it is velocity of separation.

Solved Example

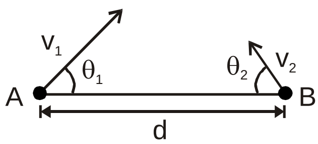
Example 24. Particle A is at rest and particle B is moving with constant velocity v as shown in the diagram at t = 0. Find their velocity of separation



Solution : $v_{BA} = v_B - v_A = v$

v_{sep} = component of v_{BA} along line AB = $v\cos\theta$

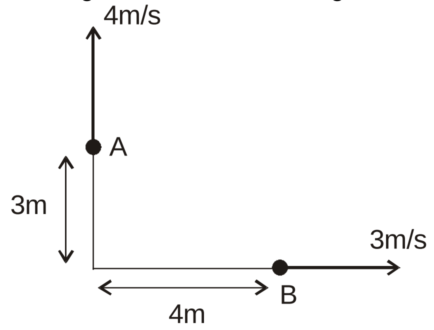
Example 25. Two particles A and B are moving with constant velocities v_1 and v_2. At t = 0, v_1 makes an angle θ_1 with the line joining A and B and v_2 makes an angle θ_2 with the line joining A and B. Find their velocity of approach.



Solution : Velocity of approach is relative velocity along line AB

$v_{APP} = v_1\cos\theta_1 + v_2\cos\theta_2$

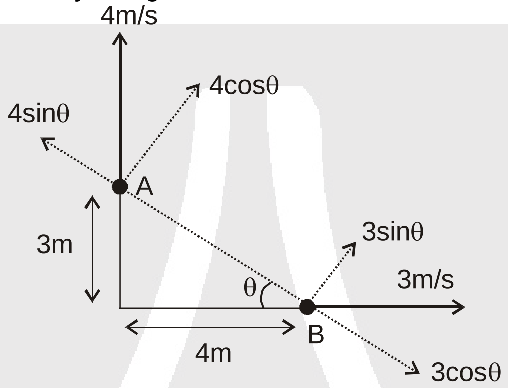
Example 26. Particles A and B are moving as shown in the diagram at t = 0. Find their velocity of separation



(i) at t = 0
(ii) at t = 1 sec.

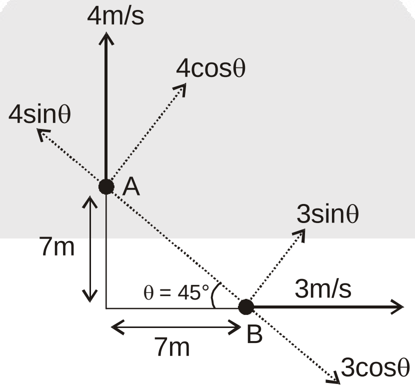
Solution :

(i) $\tan\theta = 3/4$
v_{sep} = relative velocity along line AB



$= 3\cos\theta + 4\sin\theta = 3.\ \frac{4}{5} + 4.\ \frac{3}{5} = \frac{24}{5} = 4.8$ m/s

(ii) $\theta = 45°$
v_{sep} = relative velocity along line AB



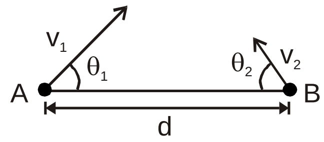
$= 3\cos\theta + 4\sin\theta = 3.\ \frac{1}{\sqrt{2}} + 4.\ \frac{1}{\sqrt{2}} = \frac{7}{\sqrt{2}}$ m/s

7.2. Condition for uniformly moving particles to collide

If two particles are moving with uniform velocities and the relative velocity of one particle w.r.t. other particle is directed towards each other then they will collide.

Solved Example

Example 27. Two particles A and B are moving with constant velocities v_1 and v_2. At t = 0, v_1 makes an angle θ_1 with the line joining A and B and v_2 makes an angle θ_2 with the line joining A and B.



(i) Find the condition for A and B to collide.

(ii) Find the time after which A and B will collide if separation between them is d at t = 0

Solution :

(i) For A and B to collide, their relative velocity must be directed along the line joining them. Therefore their relative velocity along the perpendicular to this line must be zero.
Thus $v_1 \sin\theta_1 = v_2 \sin\theta_2$.

(ii) $v_{APP} = v_1\cos\theta_1 + v_2\cos\theta_2$; $t = \dfrac{d}{v_{app}} = \dfrac{d}{v_1\cos\theta_1 + v_2\cos\theta_2}$

7.3. Minimum / Maximum distance between two particles

If the separation between two particles decreases and after some time it starts increasing then the separation between them will be minimum at the instant, velocity of approach changes to velocity of separation. (at this instant $v_{app} = 0$)

Mathematically S_{AB} is minimum when $\dfrac{dS_{AB}}{dt} = 0$

Similarly for maximum separation $v_{sep} = 0$.

Note :

- If the initial position of two particles are $\vec{r}_1$ and $\vec{r}_2$ and their velocities are $\vec{v}_1$ and $\vec{v}_2$ then shortest distance between the particles, $d_{shortest} = \dfrac{|\vec{r}_{12} \times \vec{v}_{12}|}{|\vec{v}_{12}|}$ and time after which this situation will occur, $t = -\dfrac{\vec{r}_{12}.\vec{v}_{12}}{|\vec{v}_{12}|^2}$

Solved Example

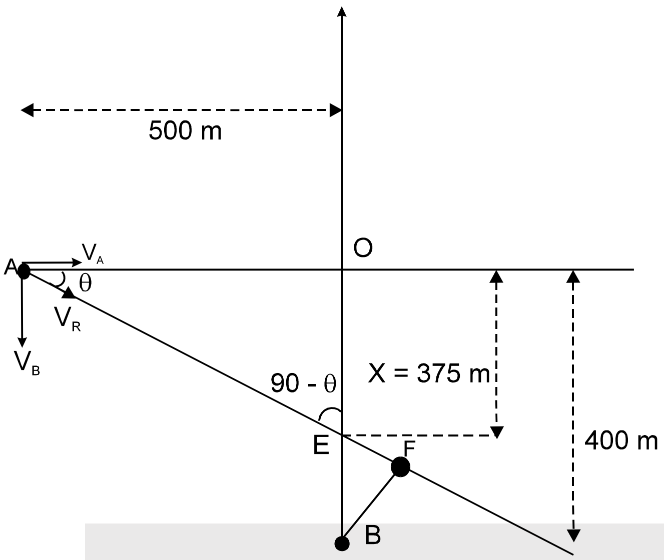
Example 28. Two cars A and B are moving west to east and south to north respectively along crossroads. A moves with a speed of 72 kmh^{-1} and is 500 m away from point of intersection of cross roads and B moves with a speed of 54 kmh^{-1} and is 400 m away from point of intersection of cross roads. Find the shortest distance between them?

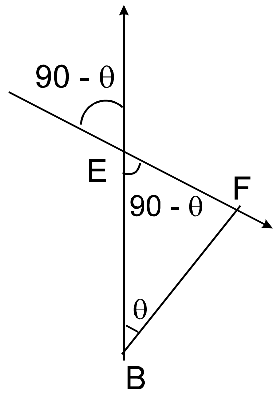
Solution :

Method 1 : (Using the concept of relative velocity)

In this method we watch the velocity of A w.r.t. B. To do this we plot the resultant velocity V_r. Since the accelerations of both the bodies is zero, so the relative acceleration between them is also zero. Hence the relative velocity will remain constant. So the path of A with respect to B will be straight line and along the direction of relative velocity of A with respect to B. The shortest distance between A & B is when A is at point F (i.e. when we drop a perpendicular from B on the line of motion of A with respect to B).

From figure





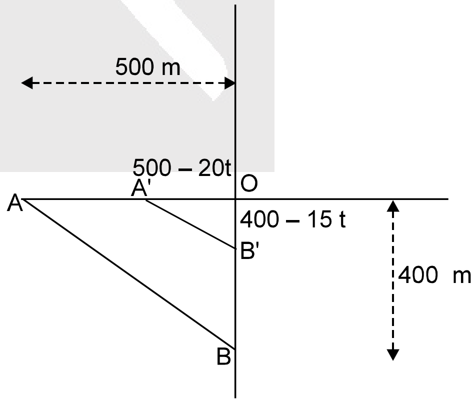
$\tan\theta = \frac{V_B}{V_A} = \frac{15}{20} = \frac{3}{4}$ ....(i)

This θ is the angle made by the resultant velocity vector with the x-axis.
Also we know that from figure

$OE = \frac{x}{500} = \frac{3}{4}$ ....(ii)

From equation (i) & (ii) we get
x = 375 m
∴ EB = OB – OE = 400 – 375 = 25 m
But the shortest distance is BF.
From magnified figure we see that BF = EB cosθ = 25 × 4/5
∴ BF = 20 m

Method 2 : (Using the concept of maxima – minima)
A & B be are the initial positions and A', B' be the final positions after time t.
B is moving with a speed of 15 m/sec so it will travel a distance of BB' = 15t during time t.
A is moving with a speed of 20 m/sec so it will travel a distance of AA' = 20t during time t.
So
OA' =500 – 20 t
OB' = 400 – 15 t
∴ $A'B'^2 = OA'^2 + OB'^2$
$= (500 - 20t)^2 + (400 - 15t)^2$ .....(i)
For A'B' to be minimum $A'B'^2$ should also be minimum



$\therefore \frac{d(A'B'^2)}{dt} = \frac{d(400-15t)^2 + (500-20t)^2}{dt} = 0$
= 2(400 – 15t) (–15) + 2(500 – 20t) (–20) = 0
= – 1200 + 45t = 2000 – 80 t
∴ 125 t = 3200
∴ $t = \frac{128}{5}$ s.

Hence A and B will be closest after $\frac{128}{5}$ s.

Now $\frac{d^2A'B'}{dt^2}$ comes out to be positive hence it is a minima.

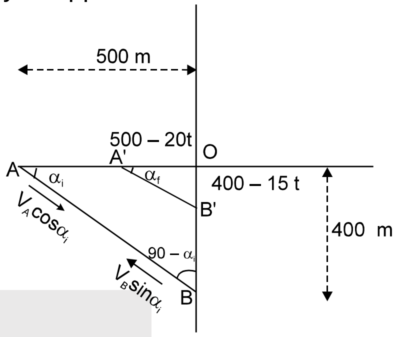
On substituting the value of t in equation (i) we get

$\therefore$ $A'B'^2 = \left(400 - 15 \times \frac{128}{5}\right)^2 + \left(500 - 20.\frac{128}{5}\right)^2 = \sqrt{16^2 + (-12)^2}$ = 20 m

$\therefore$ Minimum distance A'B' = 20 m.

Method 3 : (Using the concept of relative velocity of approach)

After time t let us plot the components of velocity of A and B in the direction along AB. When the distance between the two is minimum, the relative velocity of approach is zero.



$\therefore$ $V_A \cos\alpha_f + V_B \sin\alpha_f = 0$

(where α_f is the angle made by the line A'B' with the x-axis)

$20 \cos\alpha_f = -15 \sin\alpha_f$

$\therefore$ $\tan\alpha_f = -\frac{20}{15} = -\frac{4}{3}$

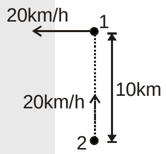
(Here do not confuse this angle with the angle θ in method (I) because that θ is the angle made by the net relative velocity with x-axis, but α_f is the angle made by the line joining the two particles with x-axis when velocity of approach in zero.)

$\therefore$ $\frac{400 - 15t}{500 - 20t} = -\frac{4}{3}$

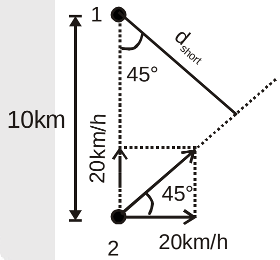
$\therefore$ $t = \frac{128}{5}$ So, OB' = 16 m and OA' = – 12m

$A'B' = \sqrt{16^2 + (-12)^2}$ = 20 m

Example 29. Two ships are 10 km apart on a line joining south to north. The one farther north is steaming west at 20 km h^{-1}. The other is steaming north at 20 km h^{-1}. What is their distance of closest approach ? How long do they take to reach it ?



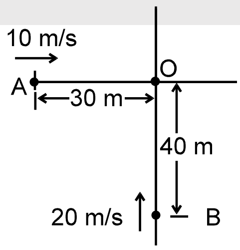
Solution : Solving from the frame of particle -1



we get $d_{short} = 10 \cos 45° = \frac{10}{\sqrt{2}} = 5\sqrt{2}$ km

$t = \frac{10 \sin 45}{|\vec{V}_{21}|} = \frac{10 \times 1/\sqrt{2}}{20\sqrt{2}} = \frac{1}{4}h$ = 15 min.

Example 30. Two particles A and B are moving with uniform velocity as shown in the figure given below at t = 0.



(i) Will the two particle collide

(ii) Find out shortest distance between two particles

Solution : Solving from the frame of B

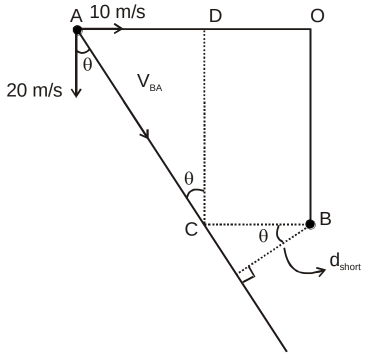
we get $\tan\theta = \frac{10}{20} = \frac{1}{2}$

again $\tan\theta = \frac{AD}{CD} = \frac{AD}{40} = \frac{1}{2}$

$\Rightarrow$ AD = 20 $\Rightarrow$ DO = 10 $\Rightarrow$ BC = 10

d_{short} = BC cosθ = 10 cosθ = $\frac{10\times 2}{\sqrt{5}} = 4\sqrt{5}$ m

Since closest distance is non zero therefore they will not collide

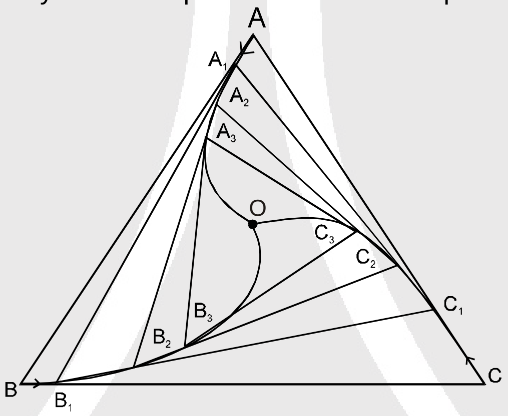


7.4. Miscellaneous Problems on collision

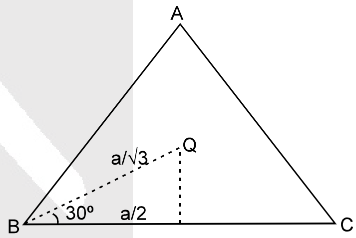
Solved Example

Example 31. There are particles A, B and C are situated at the vertices of an equilateral triangle ABC of side a at t = 0. Each of the particles moves with constant speed v. A always has its velocity along AB, B along BC and C along CA. At what time will the particle meet each other?

Solution : The motion of the particles is roughly sketched in figure. By symmetry they will meet at the centroid O of the triangle. At any instant the particles will from an equilateral triangle ABC with the same



Centroid O. All the particles will meet at the centre. Concentrate on the motion of any one particle, say B. At any instant its velocity makes angle 30º with BO. The component of this velocity along BO is v cos 30º. This component is the rate of decrease of the distance BO. Initially

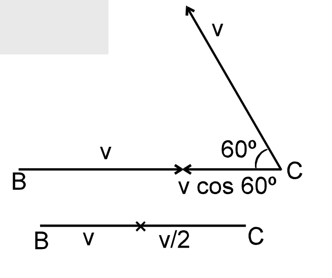


BO = $\frac{a/2}{\cos 30^o} = \frac{a}{\sqrt{3}}$ = displacement of each particle. Therefore,

the time taken for BO to become zero

= $\frac{a/\sqrt{3}}{v\cos 30^o} = \frac{2a}{\sqrt{3}v\times\sqrt{3}} = \frac{2a}{3v}$.

Aliter : Velocity of B is v along BC. The velocity of C is along CA. Its component along BC is v cos 60º = v/2. Thus, the separation BC decreases at the rate of approach velocity.

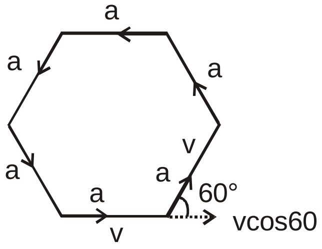


$\therefore$ approach velocity = v + $\frac{v}{2} = \frac{3v}{2}$

Since, the rate of approach is constant, the time taken in reducing the separation BC from a to zero is t = $\frac{a}{\frac{3v}{2}} = \frac{2a}{3v}$

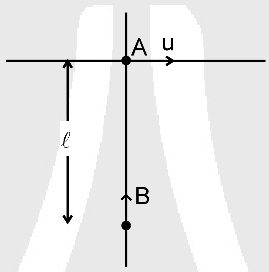
Example 32. Six particles situated at the corners of a regular hexagon of side a move at a constant speed v. Each particle maintains a direction towards the particle at the next corner. Calculate the time the particles will take to meet each other.

Solution : $V_{app} = V - V\cos 60° = V - V/2 = V/2$



$$t = \frac{a}{V_{app}} = \frac{a}{V/2} = \frac{2a}{V}$$

Example 33. 'A' moves with constant velocity u along the 'x' axis. B always has velocity towards A. After how much time will B meet A if B moves with constant speed v. What distance will be travelled by A and B.



Solution : Let at any instant the velocity of B makes an angle α with that of x axis and the time to collide is T.

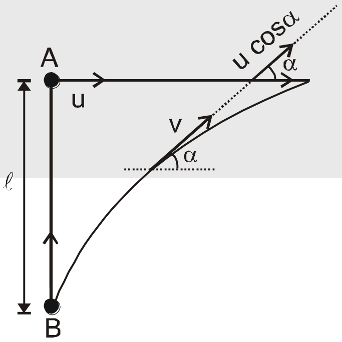
$v_{app} = v - u\cos\alpha$

$$\ell = \int_0^T v_{app}\,dt = \int_0^T (v - u\cos\alpha)dt \qquad \text{....... (1)}$$

Now equating the displacement of A and B along x direction we get

$$uT = \int v\cos\alpha\,dt \qquad \text{....... (2)}$$

Now from (1) and (2) we get



$$\ell = vT - \int_0^T u\cos\alpha\,dt = vT - \frac{u}{v}\int_0^T v\cos\alpha\,dt = vT - \frac{u}{v}.uT$$

$$\Rightarrow T = \frac{\ell v}{v^2 - u^2}$$

Now distance travelled by A and B

$$= u \times \frac{\ell v}{v^2 - u^2} \text{ and } v \times \frac{\ell v}{v^2 - u^2} = \frac{uv\ell}{v^2 - u^2} \text{ and } \frac{v^2\ \ell}{v^2 - u^2}$$

Exercise-1

✍ **Marked Questions can be used as Revision Questions.**

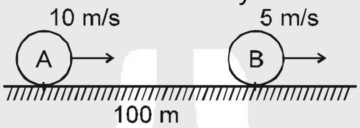
PART - I : SUBJECTIVE QUESTIONS

Section (A) : Relative motion in one dimension

A-1. Two parallel rail tracks run north-south. Train A moves due north with a speed of 54 km h^{-1} and train B moves due south with a speed of 90 km h^{-1}. A monkey runs on the roof of train A with a velocity of 18 km/h w.r.t. train A in a direction opposite to that of A. Calculate the (a) relative velocity of B with respect to A (b) relative velocity of ground with respect to B (c) velocity of a monkey as observed by a man standing on the ground. (d) Velocity of monkey as observed by a passenger of train B.

A-2. A train is moving with a speed of 40 km/h. As soon as another train going in the opposite direction passes by the window, the passenger of the first train starts his stopwatch and notes that other train passes the window in 3 s. Find the speed of the train going in the opposite direction if its length is 75 m.

A-3. An object A is moving with 10 m/s and B is moving with 5 m/s in the same direction of positive x-axis. A is 100 m behind B as shown. Find the time taken by A to meet B.



A-4. The driver of a train A running at 25 ms^{-1} sights a train B moving in the same direction on the same track with 15 ms^{-1}. The driver of train A applies brakes to produce a deceleration of 1.0 ms^{-2}. What should be the minimum distance between the trains to avoid the accident?

Section (B) : Relative motion in two dimensions

B-1. A particle A moves with a velocity $4\hat{i}$ and another particle B moves with a velocity $-3\hat{j}$. Find $\vec{V}_{AB}$, $\vec{V}_{BA}$ and their magnitude.

B-2. A ship is steaming due east at 12 ms^{-1}. A woman runs across the deck at 5 ms^{-1} (relative to ship) in a direction towards north. Calculate the velocity of the woman relative to sea.

B-3.✍ Two perpendicular rail tracks have two trains A & B respectively. Train A moves towards north with a speed of 54 km h^{-1} and train B moves towards west with a speed of 72 km h^{-1}. Assume that both trains start from same point. Calculate the
(a) Relative velocity of ground with respect to B
(b) Relative velocity of A with respect to B.

B-4. A man is swimming in a lake in a direction of 30° East of North with a speed of 5 km/h and a cyclist is going on a road along the lake shore towards East at a speed of 10 km/h. In what direction and with what speed would the man appear to swim to the cyclist.

B-5.✍ A ship is sailing towards north at a speed of $\sqrt{2}$ m/s. The current is taking it towards East at the rate of 1 m/s and a sailor is climbing a vertical pole on the ship at the rate of 1 m/s. Find the velocity of the sailor with respect to ground.

Section (C) : Relative motion in river flow & Air flow

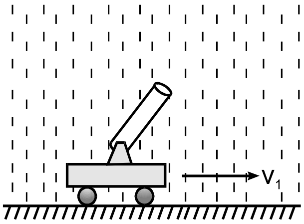
C-1. A swimmer's speed in the direction of flow of river is 16 km h^{-1}. Swimmer's speed against the direction of flow of river is 8 km h^{-1}. Calculate the swimmer's speed in still water and the velocity of flow of the river.

C-2. A man can swim with a speed of 4 km h^{-1} in still water. How long does he take to cross a river 1 km wide if the river flows steadily at 3 km h^{-1} and he makes his strokes normal to the river current? How far down the river does he go when he reaches the other bank?

C-3.✍ A river is flowing from west to east at a speed of 5 m/min. A man on the south bank of the river, capable of swimming at 10 m/min in still water, swims across the shortest path distance. In what direction should he swim?

Section (D) : Relative motion in Rain and wind

D-1. A pipe which can rotate in a vertical plane is mounted on a cart. The cart moves uniformly along a horizontal path with a speed v_1 = 2 m/s. At what angle α to the horizontal should the pipe be placed so that drops of rain falling vertically with a velocity v_2 = 6m/s move parallel to the axis of the pipe without touching its walls? Consider the velocity of the drops as constant due to the resistance of the air.



D-2. Rain seems to be falling vertically to a person sitting in a bus which is moving uniformly eastwards with 10 m/s. It appears to come from vertical at a velocity 20 m/s. Find the speed of rain drops with respect to ground.

D-3. To a man walking at the rate of 2 km/hour with respect to ground, the rain appears to fall vertically. When he increases his speed to 4 km/hour in same direction of his motion, rain appears to meet him at an angle of 45º with horizontal, find the real direction and speed of the rain.

Section (E) : Velocity of separation & approach

E-1. A particle is kept at rest at origin. Another particle starts from (5m, 0) with a velocity of $-4\hat{i} + 3\hat{j}$ m/s. Find their closest distance of approach.

E-2. Four particles situated at the corners of a square of side 'a', move at a constant speed v. Each particle maintains a direction towards the next particle in succession. Calculate the time the particles will take to meet each other.

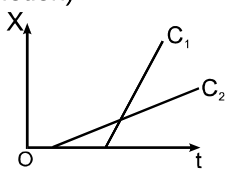
PART - II : ONLY ONE OPTION CORRECT TYPE

Section (A) : Relative motion in one dimension

A-1. An aeroplane is flying vertically upwards with a uniform speed of 500 m/s. When it is at a height of 1000 m above the ground a shot is fired at it with a speed of 700 m/s from a point directly below it. The minimum uniform acceleration of the aeroplane now so that it may escape from being hit ? (g = 10 m/s^2)
(A) 10 m/s^2 (B) 8 m/s^2 (C) 12 m/s^2 (D) None of these

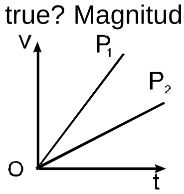
A-2. A stone is thrown upwards from a tower with a velocity 50 ms^{-1}. Another stone is simultaneously thrown downwards from the same location with a velocity 50 ms^{-1}. When the first stone is at the highest point, the relative velocity of the second stone with respect to the first stone is (assume that second stone has not yet reached the ground) :
(A) Zero (B) 50 ms^{-1} (C) 100 ms^{-1} (D) 150 ms^{-1}

A-3. Shown in the figure are the position time graph for two children going home from the school. Which of the following statements about their relative motion is true after both of them started moving?
Their relative velocity: (consider 1-D motion)



(A) first increases and then decreases (B) first decreases and then increases
(C) is zero (D) is non zero constant.

A-4. Shown in the figure are the velocity time graphs of the two particles P_1 and P_2. Which of the following statements about their relative motion is true? Magnitude of their relative velocity: (consider 1-D motion)



(A) is zero
(B) is non-zero but constant
(C) continuously decreases
(D) continuously increases

A-5. Two trains A and B which are 100 km apart are travelling towards each other on different tracks with each having initial speed of 50 km/h. The train A accelerates at 20 km/h^2 and the train B retards at the rate 20 km/h^2. The distance covered by the train A when they cross each other is :
(A) 45 km (B) 55 km (C) 65 km (D) 60 km

A-6. A jet airplane travelling from east to west at a speed of 500 km h^{-1} eject out gases of combustion at a speed of 1500 km h^{-1} with respect to the jet plane. What is the velocity of the gases with respect to an observer on the ground?
(A) 1000 km h^{-1} in the direction west to east
(B) 1000 km h^{-1} in the direction east to west
(C) 2000 km h^{-1} in the direction west to east
(D) 2000 km h^{-1} in the direction east to west

Section (B) : Relative motion in two dimension

B-1. A helicopter is flying south with a speed of 50 kmh^{-1}. A train is moving with the same speed towards east. The relative velocity of the helicopter as seen by the passengers in the train will be towards.
(A) north east (B) south east (C) north west (D) south west

B-2. Two particles are moving with velocities v_1 and v_2. Their relative velocity is the maximum, when the angle between their velocities is :
(A) zero (B) $\pi/4$ (C) $\pi/2$ (D) π

B-3. A ship is travelling due east at 10 km/h. A ship heading 30° east of north is always due north from the first ship. The speed of the second ship in km/h is -
(A) $20\sqrt{2}$ (B) 20 (C) $20\sqrt{3/2}$ (D) $20/\sqrt{2}$

Section (C) : Relative motion in river flow

C-1. A boat, which has a speed of 5 km/h in still water, crosses a river of width 1 km along the shortest possible path in 15 minutes. The velocity of the river water in km/h is -
(A) 1 (B) 3 (C) 4 (D) $\sqrt{41}$

C-2. A boat is rowed across a river (perpendicular to river flow) at the rate of 9 km/hr. The river flows at the rate of 12 km/hr. The velocity of boat in km/hr is:
(A) 14 (B) 15 (C) 16 (D) 17

C-3. A boat which can move with a speed of 5 m/s relative to water crosses a river of width 480 m flowing with a constant speed of 4 m/s. What is the time taken by the boat to cross the river along the shortest path.
(A) 80 s (B) 160 s (C) 240 s (D) 320 s

C-4. An airplane pilot sets a compass course due west and maintains an air speed of 240 km/h. After flying for $\frac{1}{2}$ h, he finds himself over a town that is 150 km west and 40 km south of his starting point. The wind velocity (with respect to ground) is :
(A) 100 km/h, 37° W of S
(B) 100 km/h, 37° S of W
(C) 120 km/h, 37° W of S
(D) 120 km/h, 37° S of W

Section (D) : Relative motion in Rain and wind

D-1. It is raining vertically downwards with a velocity of 3 km h^{-1}. A man walks in the rain with a velocity of 4 kmh^{-1}. The rain drops will fall on the man with a relative velocity of :
(A) 1 kmh^{-1} (B) 3 kmh^{-1} (C) 4 kmh^{-1} (D) 5 kmh^{-1}

D-2. A man walks in rain with a velocity of 5 kmh^{-1}. The rain drops strike at him at an angle of 45° with the horizontal. The velocity of the rain if it is falling vertically downward is :
(A) 5 kmh^{-1} (B) 4 kmh^{-1} (C) 3 kmh^{-1} (D) 1 kmh^{-1}

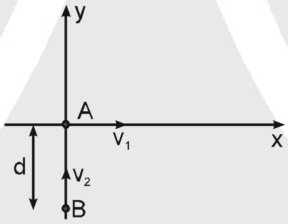
D-3. Raindrops are falling vertically with a velocity of 10 m/s. To a cyclist moving on a straight road the raindrops appear to be coming with a velocity of 20 m/s. The velocity of cyclist is :
(A) 10 m/s (B) $10\sqrt{3}$ m/s (C) 20 m/s (D) $20\sqrt{3}$ m/s

D-4. An aeroplane has to go along straight line from A to B, and back again. The relative speed with respect to wind is V. The wind blows perpendicular to line AB with speed υ. The distance between A and B is ℓ. The total time for the round trip is:
(A) $\frac{2\ell}{\sqrt{V^2-\upsilon^2}}$ (B) $\frac{2\upsilon\ell}{V^2-\upsilon^2}$ (C) $\frac{2V\ell}{V^2-\upsilon^2}$ (D) $\frac{2\ell}{\sqrt{V^2+\upsilon^2}}$

Section (E) : Velocity of separation & approach

E-1. For two particles A and B, given that $\vec{r}_A = 2\hat{i} + 3\hat{j}$, $\vec{r}_B = 6\hat{i} + 7\hat{j}$, $\vec{v}_A = 3\hat{i} - \hat{j}$ and $\vec{v}_B = x\hat{i} - 5\hat{j}$. What is the value of x if they collide?
(A) 1 (B) – 1 (C) 2 (D) –2

E-2#. Two particles A and B move with velocities v_1 and v_2 respectively along the x & y axis. The initial separation between them is 'd' as shown in the figure. Find the least distance between them during their motion.



(A) $\frac{d.v_1^2}{v_1^2+v_2^2}$ (B) $\frac{d.v_2^2}{v_1^2+v_2^2}$ (C) $\frac{d.v_1}{\sqrt{v_1^2+v_2^2}}$ (D) $\frac{d.v_2}{\sqrt{v_1^2+v_2^2}}$

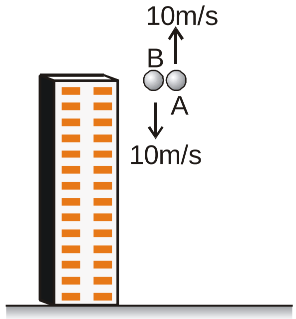
PART - III : MATCH THE COLUMN

1. Match the following:
A ball is thrown vertically upward in the air by a passenger (relative to himself) from a train that is moving as given in column I ($v_{ball} << v_{escape}$). Correctly match the situation as described in the column I, with the paths given in column II.

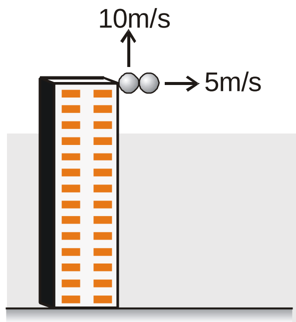
Column- I	Column-II
(A) Train moving with constant acceleration on a slope then path of the ball as seen by the passenger.	(p) Straight line
(B) Train moving with constant acceleration on a slope then path of the ball as seen by a stationary observer outside.	(q) Parabolic
(C) Train moving with constant acceleration on horizontal ground then path of the ball as seen by the passenger.	(r) Elliptical
(D) Train moving with constant acceleration on horizontal ground then path of the ball as seen by a stationary observer outside.	(s) Hyperbolic
	(t) Circular

2. Both A & B are thrown simultaneously as shown from a very high tower.

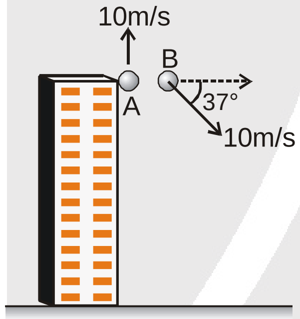
Column-I | **Column-II**

(A) 

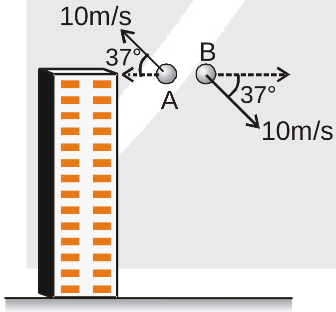
(p) Distance between the two balls at two seconds is $16\sqrt{5}$ m.

(B) 

(q) distance between two balls at 2 seconds is 40 m.

(C) 

(r) distance between two balls at 2 sec is $10\sqrt{5}$ m.

(D) 

(s) Magnitude of relative velocity of B w.r.t A is $5\sqrt{2}$ m/s.

(t) magnitude of relative velocity of B with respect to A is $5\sqrt{5}$ m/s.

Exercise-2

Marked Questions can be used as Revision Questions.

PART - I : ONLY ONE OPTION CORRECT TYPE

1. Two cars are moving in the same direction with a speed of 30 km h^{-1}. They are separated from each other by 5 km. Third car moving in the opposite direction meets the two cars after an interval of 4 minutes. What is the speed of the third car ?
(A) 35 km h^{-1} (B) 40 km h^{-1} (C) 45 km h^{-1} (D) 75 km h^{-1}

2. A bus is moving with a velocity 10 ms^{-1} on a straight road. A scooterist wishes to overtake the bus in 100s. If, the bus is at a distance of 1 km from the scooterist, with what velocity should the scooterist chase the bus? (Neglect size of the bus)
(A) 50 ms^{-1} (B) 40 ms^{-1} (C) 30 ms^{-1} (D) 20 ms^{-1}

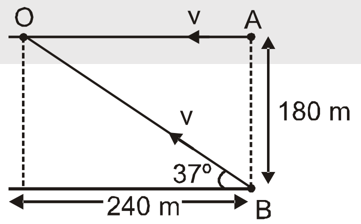
3. A coin is released inside a lift at a height of 2 m from the floor of the lift. The height of the lift is 10 m. The lift is moving with an acceleration of 11 m/s^2 downwards. The time after which the coin will strike with the lift is :
(A) 4 s (B) 2 s (C) $\frac{4}{\sqrt{21}}$ s (D) $\frac{2}{\sqrt{11}}$ s

4. A police van moving on a highway with a speed of 30 km h^{-1} fires a bullet at a thief's car speeding away in the same direction with a speed of 192 km h^{-1}. If the muzzle speed of the bullet is 150 ms^{-1}, with what speed does the bullet hit the thief's car (as, seen by thief). According to thief in the car ?
(A) 105 m/s (B) 100 m/s (C) 110 m/s (D) 90 m/s

5. A flag on a bus is fluttering in north direction & wind is blowing in east direction. Then which of the following will be true -
(A) bus is moving in south direction.
(B) bus is moving in north east direction.
(C) bus may be moving in any direction between south & east.
(D) bus may be moving in any direction between south & west.

6. For four particles A, B, C & D the velocities of one with respect to other are given as $\vec{V}_{DC}$ is 20 m/s towards north, $\vec{V}_{BC}$ is 20 m/s towards east and $\vec{V}_{BA}$ is 20 m/s towards south. Then $\vec{V}_{DA}$ is
(A) 20 m/s towards north (B) 20 m/s towards south
(C) 20 m/s towards east (D) 20 m/s towards west

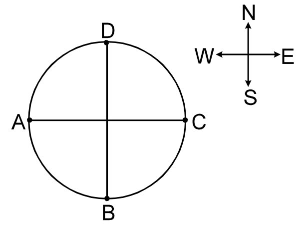
7. Two persons P and Q start from points A and B respectively as shown in figure. P and Q have speed v = 12 m/s in shown directions towards point O. when the distance between P and Q is 120m, then Q increases its speed to 15 m/s. Then find out who will reach the point O first.



(A) P (B) Q
(C) both P and Q reaches simultaneously (D) Data is insufficient

8. A man crosses the river perpendicular to river flow in time t seconds and travels an equal distance down the stream in T seconds. The ratio of man's speed in still water to the speed of river water will be :
(A) $\frac{t^2 - T^2}{t^2 + T^2}$ (B) $\frac{T^2 - t^2}{T^2 + t^2}$ (C) $\frac{t^2 + T^2}{t^2 - T^2}$ (D) $\frac{T^2 + t^2}{T^2 - t^2}$

9. Two aeroplanes fly from their respective positions 'A' and 'B' starting at the same time and reach the point 'C' simultaneously when wind was not blowing. On a windy day they head towards 'C' but both reach the point 'D' simultaneously in the same time which they took to reach 'C'. Then the wind is blowing in



(A) North-East direction
(B) North-West direction
(C) Direction making an angle $0 < \theta < 90$ with North towards East.
(D) North direction

10. A man who is wearing a hat of extended length of 12 cm is running in rain falling vertically downwards with speed 10 m/s. The maximum speed with which man can run, so that rain drops do not fall on his face (the length of his face below the extended part of the hat is 16 cm) will be:

(A) $\frac{15}{2}$ m/s (B) $\frac{40}{3}$ m/s (C) 10 m/s (D) zero

11. A man is going up in a lift (open at the top) moving with a constant velocity 3 m/s. He throws a ball up at 5 m/sec relative to the lift when the lift is 50 m above the ground. Height of the lift when the ball meets it during its downward journey is ($g = 10\ m/s^2$) **[Olympiad (Stage-1) 2017]**

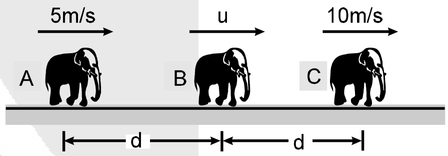
(A) 53 m (B) 58 m (C) 63 m (D) 68 m

PART - II : SINGLE AND DOUBLE VALUE INTEGER TYPE

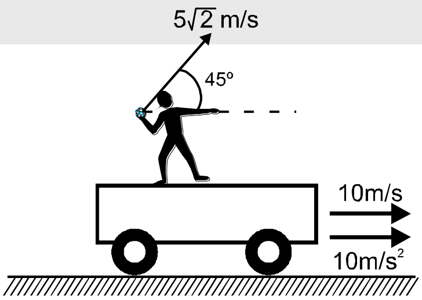
1. Men are running along a road at 15 km/h behind one another at equal intervals of 20 m. Cyclists are riding in the same direction at 25 km/h at equal intervals of 30 m. At what speed (in km/h) an observer travel along the road in opposite direction so that whenever he meets a runner he also meets a cyclist? (Neglect the size of cycle)

2. Two identical trains take 3 sec to pass one another when going in the opposite direction but only 2.5 sec if the speed of one is increased by 50%. Find the time (in sec) one would take to pass the other when going in the same direction at their original speed.

3. Three elephants A, B and C are moving along a straight line with constant speed in same direction as shown in figure. Speed of A is 5 m/s and speed of C is 10 m/s. Initially separation between A & B is 'd' and between B & C is also d. When 'B' catches 'C' separation between A & C becomes 3d. Find the speed of B (in m/s).



4. A man standing on a truck which moves with a constant horizontal acceleration a (= 10 m/s²) when speed of the truck is 10 m/s. The man throws a ball with velocity $5\sqrt{2}$ m/s with respect to truck. In the direction shown in the diagram. Find the displacement of ball in meters in one second as observed by the man. ($g = 10\ m/s^2$)



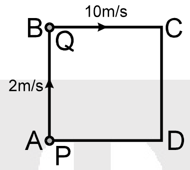
5. A swimmer crosses the river along the line making an angle of 45° with the direction of flow. Velocity of the river water is 5 m/s. Swimmer takes 6 seconds to cross the river of width 60 m. If the velocity of the swimmer with respect to water is $5\sqrt{n}$ m/s, then find n.

6.✍ An aeroplane has to go from a point A to another point B, 1000 km away due 30º west of north. A wind is blowing due north at a speed of 20 m/s. The air-speed of the plane is 150 m/s. If the angle at which the pilot should head the plane to reach the point B is $\sin^{-1}(1/n)$ west of the line AB, Then find n.

7.✍ Rain appears to be falling at an angle of 37° with vertical to the driver of a car moving with a velocity of 7 m/sec. When he increases the velocity of the car to 25 m/sec, the rain again appears to fall at an angle 37° with vertical. If the actual velocity of rain relative to ground is 4n m/s then find n.

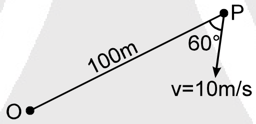
8.✍ During a rainy day, rain is falling vertically with a velocity 2m/s. A boy at rest starts his motion with a constant acceleration of $2 m/s^2$ along a straight road. If the rate at which the angle of the axis of umbrella with vertical should be changed is 1/n at t = 5s so that the rain falls parallel to the axis of the umbrella, then find n.

9. Two men P & Q are standing at corners A & B of square ABCD of side 8 m. They start moving along the track with constant speed 2 m/s and 10 m/s respectively. Find the time (in seconds) when they will meet for the first time.



10. Two straight tracks AOB and COD meet each other at right angles at point O. A person walking at a speed of 5 km/h along AOB is at the crossing O at 12 o'clock noon. Another person walking at the same speed along COD reaches the crossing O at 1:30 PM. If the time at which the distance between them is least is 12 : T PM, then find T.

11. P is a point moving with constant speed 10 m/s such that its velocity vector always maintains an angle 60° with line OP as shown in figure (O is a fixed point in space). The initial distance between O and P is 100 m. After what time (in sec) shall P reach O.



PART - III : ONE OR MORE THAN ONE OPTIONS CORRECT TYPE

1. A man standing on the edge of the terrace of a high rise building throws a stone vertically up with a speed of 20 m/s. Two seconds later an identical stone is thrown vertically downwards with the same speed of 20 m/s. Then :
(A) the relative velocity between the two stones remain constant till one hits the ground
(B) both will have the same kinetic energy when they hit the ground
(C) the time interval between their hitting the ground is 2 seconds
(D) if the collisions on the ground are perfectly elastic both will rise to the same height above the ground.

2.✍ A man in a lift which is ascending with an upward acceleration 'a' throws a ball vertically upwards with a velocity 'v' with respect to himself and catches it after 't_1' seconds. Afterwards when the lift is descending with the same acceleration 'a' acting downwards the man again throws the ball vertically upwards with the same velocity with respect to him and catches it after 't_2' seconds
(A) the acceleration of the ball with respect to ground is g when it is in air
(B) the velocity v of the ball relative to the lift is $\frac{g\,(t_1+t_2)}{t_1\,t_2}$
(C) the acceleration 'a' of the lift is $\frac{g\,(t_2-t_1)}{t_1+t_2}$
(D) the velocity 'v' of the ball relative to the man is $\frac{g\,t_1\,t_2}{(t_1+t_2)}$

3. At an instant particle-A is at origin and moving with constant velocity $(3\hat{i}+4\hat{j})$ m/s and particle-B is at (4, 4)m and moving with constant velocity $(4\hat{i}-3\hat{j})$ m/s. Then :
(A) at this instant relative velocity of B w.r.t. A is $(\hat{i}-7\hat{j})$ m/s
(B) at this instant approach velocity of A and B is $3\sqrt{2}$ m/s
(C) relative velocity of B w.r.t. A remains constant
(D) approach velocity of A and B remains constant

4. A person is standing on a truck moving with a constant velocity of 15 m/s on a horizontal road. The man throws a ball in such a way that it returns to his hand after the truck has moved 60 m. (g = 10 m/s^2)
(A) The speed of the ball as seen from the truck is 20 m/s
(B) The direction of initial velocity of ball is upward as seen from the truck
(C) The initial speed of the ball as seen from the ground is 25 m/s
(D) None of these

5. Two boats A and B having same speed relative to river are moving in a river. Boat A moves normal to the river current as observed by an observer moving with velocity of river current. Boat B moves normal to the river as observed by the observer on the ground. Choose the **incorrect** options.
(A) To a ground observer boat B moves faster than A
(B) To a ground observer boat A moves faster than B
(C) To the given moving observer boat B moves faster than A
(D) To the given moving observer boat A moves faster than B

6. An open elevator is ascending with zero acceleration and speed 10 m/s. A ball is thrown vertically up by a boy (boy is in elevator) when he is at a height 10 m from the ground, the velocity of projection is 30m/s with respect to elevator. Choose correct option(s) assuming height of the boy very small : (g = 10 m/s^2)
(A) Maximum height attained by the ball from ground is 90 m.
(B) Maximum height attained by the ball with respect to lift from the point of projection is 45 m.
(C) Time taken by the ball to meet the elevator again is 6 sec
(D) The speed of the ball when it comes back to the boy is 20 m/s with respect to ground.

PART - IV : COMPREHENSION

Comprehension # 1

The driver of a car travelling at a speed of 20 m/s, wishes to overtake a truck that is moving with a constant speed of 20 m s^{-1} in the same lane. The car's maximum acceleration is 0.5 m s^{-2}. Initially the vehicles are separated by 40 m, and the car returns back into its lane after it is 40 m ahead of the truck. The car is 3 m long and the truck 17m long.

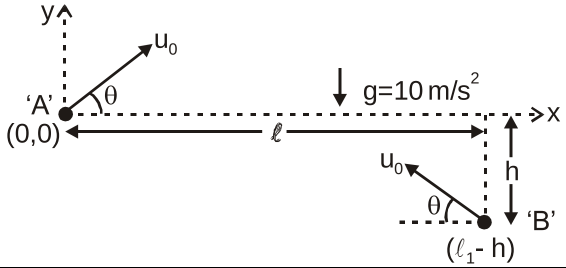
1. Find the minimum time required for the car to pass the truck and return back to its lane?
(A) 10 second (B) 20 second, (C) 15 second (D) none of these.

2. What distance does the car travel during this time?
(A) 500 m (B) 600 m (C) 200 m (D) 300 m

3. What is the final speed of the car ?
(A) 40 m/s (B) 20 m/s (C) 45 m/s (D) 30 m/s

Comprehension # 2

Two particles 'A' and 'B' are projected in the vertical plane with same initial speed u_0 from position (0, 0) and (ℓ, – h) towards each other as shown in figure at t = 0.



4. The path of particle 'A' with respect to particle 'B' will be
(A) parabola (B) straight line parallel to x-axis.
(C) straight line parallel to y-axis (D) none of these.

5. Minimum distance between particle A and B during motion will be :
(A) ℓ (B) h (C) $\sqrt{\ell^2 + b^2}$ (D) ℓ + h

6. The time when separation between A and B is minimum is :
(A) $\frac{x}{u_0 \cos\theta}$ (B) $\sqrt{\frac{2h}{g}}$ (C) $\frac{\ell}{2u_0 \cos\theta}$ (D) $\frac{2\ell}{u_0 \cos\theta}$

Comprehension # 3

Raindrops are falling with a velocity $10\sqrt{2}$ m/s making an angle of 45° with the vertical. The drops appear to be falling vertically to a man running with constant velocity. The velocity of rain drops change such that the rain drops now appear to be falling vertically with $\sqrt{3}$ times the velocity it appeared earlier to the same person running with same velocity.

7. The magnitude of velocity of man with respect to ground is
(A) $10\sqrt{2}$ m/s (B) 5 m/s (C) 20 m/s (D) 10 m/s

8. After the velocity of rain drops change, the magnitude of velocity of raindrops with respect to ground is
(A) 20 m/s (B) 25 m/s (C) 10 m/s (D) 15 m/s

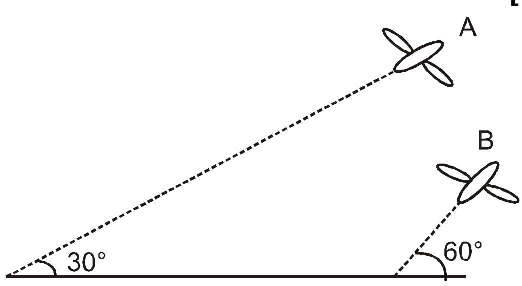
9. The angle (in degrees) between the initial and the final velocity vectors of the raindrops with respect to the ground is
(A) 8 (B) 15 (C) 22.5 (D) 37

Exercise-3

✍ Marked Questions can be used as Revision Questions.
*** Marked Questions may have more than one correct option.**

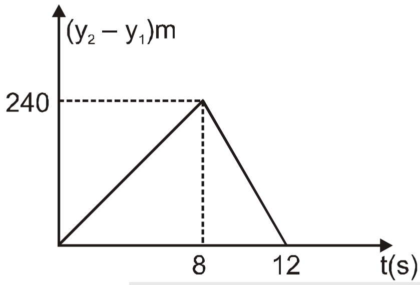
PART - I : JEE (ADVANCED) / IIT-JEE PROBLEMS (PREVIOUS YEARS)

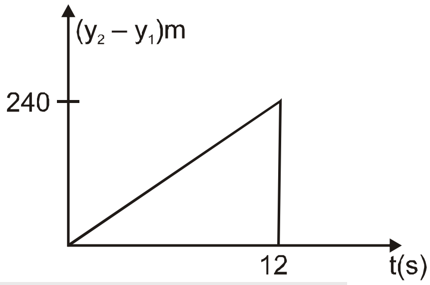
1. Airplanes A and B are flying with constant velocity in the same vertical plane at angles 30° and 60° with respect to the horizontal respectively as shown in figure. The speed of A is $100\sqrt{3}$ ms^{-1}. At time t = 0s, an observer in A finds B at a distance of 500m. This observer sees B moving with a constant velocity perpendicular to the line of motion of A. If at t = t_0, A just escapes being hit by B, t_0 in seconds is:
[JEE (Advanced) 2014; P-1, 3/60]

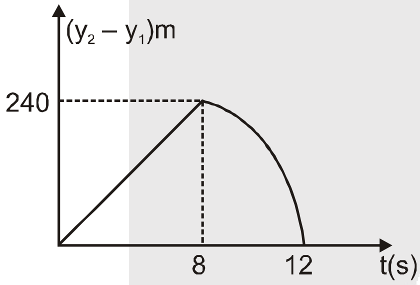


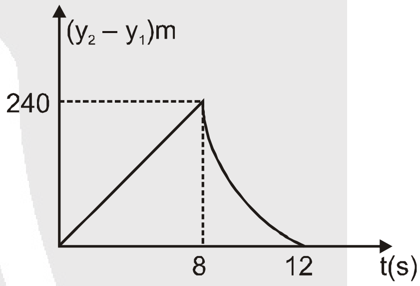
PART - II : JEE (MAIN) / AIEEE PROBLEMS (PREVIOUS YEARS)

1. Two stones are thrown up simultaneously from the edge of a cliff 240 m high with initial speed of 10 m/s and 40 m/s respectively. Which of the following graph best represents the time variation of relative position of the second stone with respect to the first ? (Assume stones do not rebound after hitting the ground and neglect air resistance, take g = 10 m/s^2) (The figures are schematic and not drawn to scale.) **[JEE (Main) 2015; 4/120, –1]**

(1)


(2)


(3)


(4)


2. Ship A is sailing towards north-east with velocity $\vec{v} = 30\hat{i} + 50\hat{j}$ km/hr where $\hat{i}$ points east and $\hat{j}$ points north. Ship B is at a distance of 80 km east and 150 km north of ship A and is sailing towards west at 10 km/hr. A will be at minimum distance from B in. **[JEE (Main) 2019; 4/120, –1]**

(1) 4.2 hrs. (2) 2.6 hrs. (3) 3.2 hrs. (4) 2.2 hrs.

3. A particle is moving along the x-axis with its coordinate with time t given by x(t) = 10 + 8t – 3t^2. Another particle is moving along the y-axis with its coordinate as a function of time given by y(t) = 5 – 8t^3. At t = 1s, the speed of the second particle as measured in the frame of the first particle is given as $\sqrt{v}$. Then v (in m/s) is. **[JEE (Main) 2020, 08 January; 4/100, –1]**

Answers

EXERCISE-1
PART - I

Section (A) :

A-1. (a) 144 km/h due south
(b) 90 km/h due north
(c) 36 km/h due north
(d) 126 km/h due north

A-2. 50 km/h **A-3.** 20 sec.

A-4. 50 m

Section (B) :

B-1. $4\hat{i} + 3\hat{j}$, $-4\hat{i} - 3\hat{j}$, 5 unit, 5 unit.

B-2. 13 m/s, $\tan^{-1}\left(\frac{5}{12}\right)$ = 22°37′ north of east

B-3. (a) 20 m/s or 72 km/h due east
(b) 25 m/s or 90 km/h at 37°N of E

B-4. 30º N of W at $5\sqrt{3}$ km/h.

B-5. $\hat{i} + \sqrt{2}\hat{j} + \hat{k}$, $\hat{i} \rightarrow$ east, $\hat{j} \rightarrow$ north,
$\hat{k} \rightarrow$ vertical upward

Section (C) :

C-1. 12 km/h, 4km/h **C-2.** $\frac{1}{4}$ h, $\frac{3}{4}$ km

C-3. At an angle 30° west of north

Section (D) :

D-1. $\alpha = \tan^{-1} 3$ **D-2.** $10\sqrt{5}$ m/s

D-3. $2\sqrt{2}$ m/s, 45º with vertical and away from the man.

Section (E) :

E-1. 3 m **E-2.** a/v

PART - II

Section (A) :

A-1.	(A)	**A-2.**	(C)	**A-3.**	(D)
A-4.	(D)	**A-5.**	(D)	**A-6.**	(A)

Section (B) :

B-1.	(D)	**B-2.**	(D)	**B-3.**	(B)

Section (C) :

C-1.	(B)	**C-2.**	(B)	**C-3.**	(B)
C-4.	(A)				

Section (D) :

D-1.	(D)	**D-2.**	(A)	**D-3.**	(B)
D-4.	(A)				

Section (E) :

E-1. (B) **E-2.** (C)

PART - III

1. (A) – q ; (B) – q ; (C) – q ; (D) – q
2. (A) – q ; (B) – r, t ; (C) – p ; (D) – q

EXERCISE-2
PART - I

1.	(C)	**2.**	(D)	**3.**	(A)
4.	(A)	**5.**	(C)	**6.**	(D)
7.	(A)	**8.**	(C)	**9.**	(B)
10.	(A)	**11.**	(A)		

PART – II

1.	5	**2.**	15	**3.**	15
4.	0	**5.**	5	**6.**	15
7.	5	**8.**	26	**9.**	3
10.	45	**11.**	20		

PART - III

1.	(ABCD)	**2.**	(ACD)	**3.**	(ABC)
4.	(ABC)	**5.**	(ACD)	**6.**	(ABCD)

PART - IV

1.	(B)	**2.**	(A)	**3.**	(D)
4.	(B)	**5.**	(B)	**6.**	(C)
7.	(D)	**8.**	(A)	**9.**	(B)

EXERCISE-3
PART - I

1. 5

PART - II

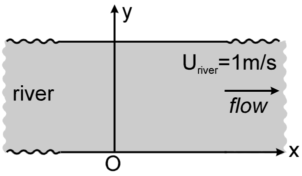
1.	(3)	**2.**	(2)	**3.**	580

High Level Problems (HLP)

Marked Questions can be used as Revision Questions.

SUBJECTIVE QUESTIONS

1. A man can swim in still water with a speed of 3 m/s. x and y axis are drawn along and normal to the bank of river flowing to right with a speed of 1 m/s. The man starts swimming from origin O at t = 0 second. Assume size of man to be negligible. Find the equation of locus of all the possible points where man can reach at t = 1 sec.



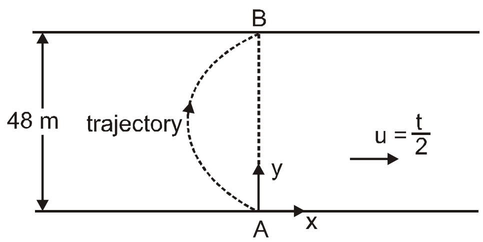
2. Two swimmers 'A' & 'B', one located on one side and other on the another side of a river are situated at a distance 'D' from each other. Line joining them is making an angle θ with the direction perpendicular to flow. The speed of each swimmer with respect to still water is 'u' & speed of river flow is v_r. Both A & B start swimming at the same time in the direction parallel to line AB towards each other and they keep on swimming in same direction. Then,
(a) Find the time after which they will meet.
(b) Find the speed of river (v_r) so that the path of the two swimmers with respect to the ground becomes perpendicular to each other.

3. Two particles start simultaneously from the same point and move along two straight lines making an angle α with each other. One move with uniform velocity u and the other with constant acceleration a and initial velocity zero.
(a) Find the least relative velocity of one with respect to other.
(b) At the same time, find the distance between the two particles.

4. A train of length ℓ = 350 m starts moving rectilinearly with constant acceleration $\omega = 3.0 \times 10^{-2}$ m/s^2; t = 30 s after the start the locomotive headlight is switched on (event 1), and τ = 60 sec after that event the train signal light is switched on (event 2). Find the distance between these events in the reference frames fixed to the train and to the Earth. How and at what constant velocity V relative to the Earth must a certain reference frame K move for the two events to occur in it at the same point?

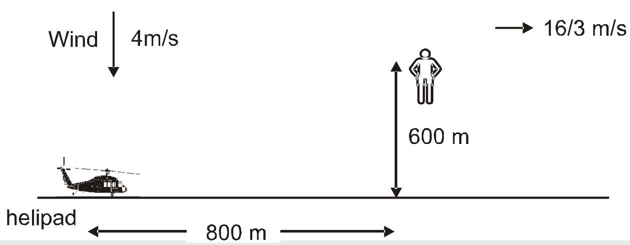
5. Find the time an aeroplane having velocity v (relative to air), takes to fly around a square with side a and the wind blowing at a velocity u, in the two cases,
(a) if the direction of wind is along one side of the square,
(b) If the direction of wind is along one of the diagonals of the square

6. A man starts swimming at time t = 0 from point A on the ground and he wants to reach the point B directly opposite the point A. His velocity in still water is 5 $\frac{m}{sec}$ and width of river is 48 m. River flow velocity 'u' varies with time t (in seconds) as u = $\frac{t}{2}\frac{metre}{sec}$. He always tries to swim in particular fixed direction with river flow. Find the (Given $\sin^{-1}\left(\frac{24}{25}\right)$ = 74º)



(a) direction (with line AB) in which he should make stroke and the time taken by man to cross the river.
(b) trajectory of path.

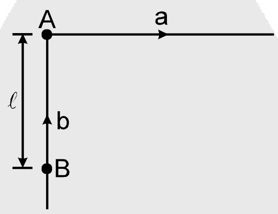
7. A child in danger of drowing in a river is being carried downstream by a current that flows uniformly at a speed of $\frac{16}{3}$ m/sec. The child is 600m from shore and 800m downstream of a helipad when the rescue helicopter sets out and the helicopter follows the same direction throughout the motion. If the helicopter proceeds at its maximum speed of $\frac{80}{3}$ m/sec. with respect to air and air is blowing with velocity of 4m/sec perpendicular to river flow velocity as shown, then



(i) Heading at what angle with the shore should the helicopter take off?
(ii) Calculate the time taken by helicopter to reach the child.

8. A boy sitting at the rear end of a railway compartment of a train, running at a constant acceleration on horizontal rails, throws a ball towards the fore end of the compartment with a muzzle velocity of 20 m/sec at an angle 37º above the horizontal, when the train is running at a speed of 10 m/sec. If the same boy catches the ball without moving from his seat and at the same height of projection, find the speed of the train at the instant of his catching the ball. [$g = 10$ m/sec^2; sin 37º = 3/5]

9. In the figure shown A and B are two particles which start from rest. A has constant acceleration ' a ' in the direction shown. B also increases its speed at a constant rate ' b ', but the direction of velocity is always towards A. Find the time after which B meets A. Also find the total distance travelled by B. ($b > a$)

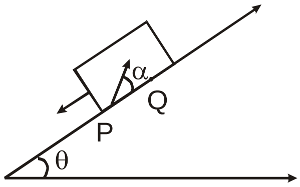


10. Two swimmers start from point A on one bank of a river to reach point B on the other bank, lying directly opposite to point A. One of them crosses the river along the straight line AB, while the other swims at right angles to the stream and then walks the distance which he has been carried away by the stream to get to point B. What was the velocity (assumed uniform) of his walking if both the swimmers reached point B simultaneously? Velocity of each swimmer in still water is 2.5 km h^{-1} and the stream velocity is 2 km h^{-1}.

11. A ship moves along the equator to the east with velocity v_0 = 30 km/hour. The southeastern wind blows at an angle ϕ = 60° to the equator with velocity v = 15 km/hour. Find the wind velocity v' relative to the ship and the angle ϕ' between the equator and the wind direction in the reference frame fixed to the ship.

12. Two cars A and B are racing along straight line. Car A is leading, such that their relative velocity is directly proportional to the distance between the two cars. When the lead of car A is ℓ_1 = 10 m, it's running 10 m/s faster than car B. If the time car A will take to increase its lead to ℓ_2 = 20 m from car B is t = ($\log_e$n) sec, then find n.

13. A large heavy box is sliding without friction down a smooth plane of inclination θ from a point P on the bottom of the box, a particle is thrown inside box. The initial speed of the particle with respect to the box is u and the direction of projection makes an angle α with the bottom as shown in the figure : **[JEE-1998, 5 + 3/120]**



(a) Find the distance along the bottom of the box between the point of projection P and the point Q where the particle lands (Assume that the particle does not hit any other surface of the box. Neglect air resistance.)

(b) If the horizontal displacement of the particle as seen by an observer on the ground is zero, find the speed of the box with respect to the ground at the instant when the particle was projected.

14. A motorboat going downstream overcame a raft (A wooden block) at a point A; τ = 60 min later it turned back and after some time passed the boat meets raft at a distance ℓ = 6.0 km from the point A. Find the flow velocity assuming the duty of the engine to be constant.

15. A boat moves relative to water with a velocity half of the river flow velocity. If the angle from the direction of flow at which the boat must move relative to stream direction to minimize drift is $\frac{2\pi}{n}$, then find n.

HLP Answers

1. $(x-1)^2 + y^2 = 9$

2. (a) $\frac{D}{2u}$ (b) $v_r = u$

3. (a) $(v_{rel})_{least} = u \sin\alpha$ (b) separation = $\frac{u^2\cos\alpha}{2a}\sqrt{1+3\sin^2\alpha}$

4. $x_1 - x_2 = \ell - w\tau(t + \tau/2) = \frac{6}{25}$ km, Towards the train with velocity v = 4 m/s

5. (a) $\frac{2a}{v^2-u^2}\left(v+\sqrt{v^2-u^2}\right)$ (b) $2\sqrt{2}\,a\left(\frac{\sqrt{2v^2-u^2}}{v^2-u^2}\right)$

6. (a) 37º and 53º, 12 sec. and 16 sec. (b) $x = y^2/64 - \frac{3y}{4}$

7. (i) 37º (ii) 50 sec

8. 42 m/s

9. $t = \sqrt{\frac{2\ell b}{b^2-a^2}}$, $\frac{b^2\ell}{b^2-a^2}$

10. 3 km/h towards B

11. $v' = \sqrt{v_0^2+v^2+2v_0v\cos\phi} \approx 40$ km per hour, $\phi' = 19°$

12. 2

13. (a) $PQ = (u^2 \sin 2\alpha)/g\cos\theta$ (b) $v = \frac{u\cos(\alpha+\theta)}{\cos\theta}$

14. $v_B = \ell/2\tau = 3.0$ km per hour

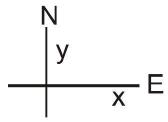
15. 3

SOLUTIONS OF REALTIVE MOTION

EXERCISE # 1

PART - I

A-1. $\vec{V}_A = 15 m/sec\,\hat{j}$ $\qquad \vec{V}_B = 25 m/sec(-\hat{j})$



(a) $\vec{V}_B - \vec{V}_A = -40\hat{j}$ i.e., 40 m/sec due south = 144 km/hr due south

(b) $0 - \vec{V}_B$ = 25 m/sec $\hat{j}$ i.e., 25 m/sec due north = 90 km/hr due north

(c) $\vec{V}_{M.G} = 15\hat{j} - \frac{18\times5}{18}\hat{j} = 10\hat{j}$ i.e., 10 m/sec due north. = 36 km/hr due north

(d) $\vec{V}_{M.B} = 10\hat{j} - (-25)\hat{j} = 35\hat{j}$ i.e., 35 m/sec due north. = 126 km/hr due north

A-2. d = $\upsilon_r t$ where υ_r is the relative velocity

$\frac{3}{60\times60} = \frac{0.075}{40+v}$ $\qquad$ 120 + 3v = 270 $\qquad$ 3v = 150 $\qquad$ v = 50 $\frac{km}{h}$

A-3. Relative velocity of A with respect to B

$\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$ = 10 – 5 = 5 m/s

So time taken by A to meet B is –

t = $\frac{100}{v_{AB}}$ = $\frac{100}{5}$ = 20 sec.

A-4. v_r = 25 – 15 = 10m/s and a_r = $-1 m/s^2$ so by $v^2 = u^2 + 2as$

S = $\frac{100}{2\times1} = 50$m

B-1. $\vec{v}_A = 4\hat{i}$, $\vec{v}_B = -3\hat{j}$

$\vec{v}_{AB} = \vec{v}_A - \vec{V}_B$ = $4\hat{i} - (-3\hat{j}) = 4\hat{i} + 3\hat{j}$

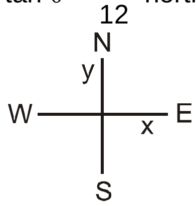
$\vec{v}_{BA} = \vec{v}_B - \vec{V}_A$ = $-3\hat{j} - 4\hat{i}$

$|\vec{v}_{AB}|$ = $\sqrt{4^2+3^2}$ = 5 unit

$|\vec{v}_{BA}| = \sqrt{3^2+4^2}$ = 5 unit.

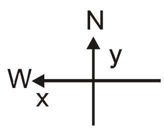
B-2. $\vec{V} = 12\hat{i} + 5\hat{j}$ $\qquad |\vec{V}| = \sqrt{12^2+5^2}$ = 13 m/sec

tan θ = $\frac{5}{12}$ north of east

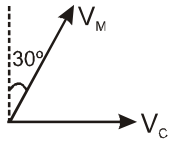


B-3. (a) $V_{G,B}$ = 0 – 20 = – 20 m/sec i.e., due east

(b) V_A = 15 m/sec $\qquad V_B$ = 20 m/sec $\qquad |\vec{V}_A - \vec{V}_B| = \sqrt{20^2+15^2}$



B-4.

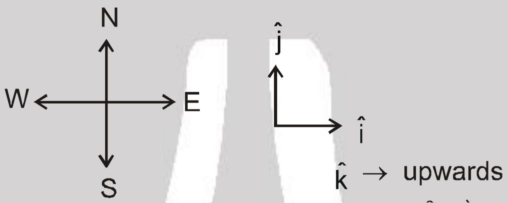


$\vec{V}_M - \vec{V}_C$ = 5 sin 30 $\hat{i}$ + 5 cos 30° $\hat{j}$ – 10 $\hat{i}$ $\quad \vec{V}_{MC} = \frac{5\sqrt{3}}{2}\hat{j} - 7.5\hat{i}$

Speed = $|\vec{V}_M - \vec{V}_C| = \sqrt{\frac{25\times3}{4} + \frac{225}{4}} = 5\sqrt{3}$ km/hr **Ans**

$\tan\theta = \frac{\frac{5\sqrt{3}}{2}}{7.5} = \frac{1}{\sqrt{3}}$; direction θ = 30° North of west. **Ans**

B-5. $V_{ship} = \sqrt{2}\,\hat{j} + 1\hat{i} + 1\hat{k} = \hat{i} + \sqrt{2}\,\hat{j} + \hat{k}$



C-1. $V_S + V_r = 16 \qquad V_S - V_r = 8$

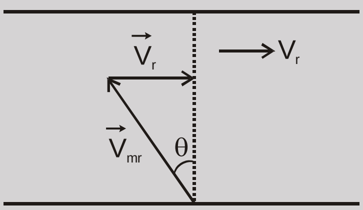
$\Rightarrow \quad V_S = 12$ km/hr $\quad \Rightarrow \quad V_r = 4$ km/hr

C-2. (I) (a) $\frac{d}{\upsilon_{mR}} = \frac{1}{4}$hr

(b) Displacement along the river flow = $\upsilon_r t = \frac{3}{4}$ km

C-3. Given $\vec{V}_r$ = 5 m/min

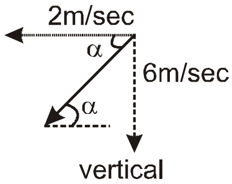
$\vec{V}_{mr}$ = 10 m/min



$\sin\theta = \frac{\vec{V}_r}{\vec{V}_{mr}} = \frac{5}{10} = \frac{1}{2}$

$\sin\theta = \frac{1}{2} \quad \Rightarrow \quad \theta = 30°$ (west of north)

D-1.

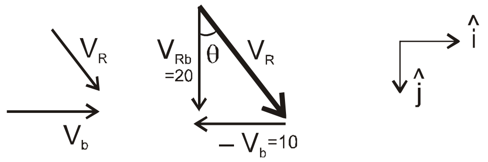


$\tan\alpha = 6/2 = 3 \qquad \alpha = \tan^{-1} 3$

D-2. $V_{RM} = V_R - V_M$

$V_R = V_{Rb} + V_b = 20\,\hat{j} + 10\,\hat{i}$

$\tan\theta = \frac{10}{20} = \frac{1}{2}$



$|V_r| = \sqrt{(20)^2 + (10)^2} = 10\sqrt{5}$ m/s

Making angle $\tan^{-1} 1/2$ with vertical.

D-3. $\vec{V}_m = 2\hat{i}$

$\vec{V}_r = \upsilon_x \hat{i} + \upsilon_y \hat{j}$

$\vec{\upsilon}_{r,m} = (\upsilon_x - 2)\hat{i} + \upsilon_y \hat{j} = \upsilon_y \hat{j}$

$\therefore \quad \upsilon_x = 2$ m/sec

$\vec{v}'_m = 4\hat{i}$

$\vec{\upsilon}'_{r,m} = \vec{\upsilon}_r - \vec{\upsilon}'_m$

$= (\upsilon_x - 4)\hat{i} + \upsilon_y \hat{j} = -2\hat{i} + \upsilon_y \hat{j}$

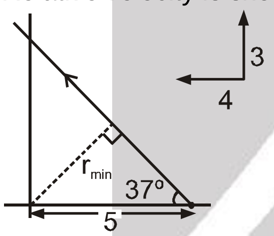
$\tan 45^\circ = \upsilon_y/2$

$\upsilon_y = 2$

so $\vec{\upsilon}_r = 2\hat{i} + 2\hat{j}$ so $\tan\theta = 1 \Rightarrow \theta = 45^\circ$

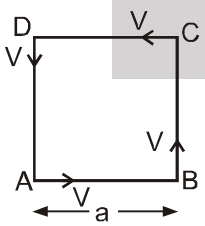
$\upsilon_r = 2\sqrt{2}$ m/sec.

E-1. Relative velocity is shown



$r_{min} = 5 \sin 37^\circ = 3$m.

E-2.



Approach velocity of A towards B = v

So, time taken = a/v.

PART - II

A-1. Acceleration of shell with respect to plane = g + a (downward)
and speed = 700 – 500 = 200 (upward)
To just escape from being hit

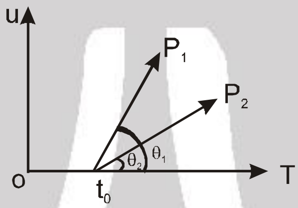
$h > \dfrac{u_{rel}^2 - v_{rel}^2}{2(g+a)} \quad \Rightarrow \quad 1000 > \dfrac{(200)^2 - (0)^2}{2(g+a)}$

$g + a > 20 \quad \Rightarrow \quad a > 10\ m/s^2$

A-2. $\upsilon_1 = 50 - gT \qquad \upsilon_2 = -50 - gT \quad v_r = \upsilon_1 - \upsilon_2 = 100$ m/sec

A-3. υ_1 = slope of C_1 line = constant
υ_2 = slope of C_2 line = constant
$\upsilon_1 - \upsilon_2 \neq 0$ but constant

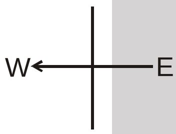
A-4. Slope of $\upsilon - t$ graph = acceleration



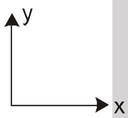
$\upsilon_1 = a_1 (t - t_0) = \tan\theta_1 t - \tan\theta_1 t_0$
$\upsilon_2 = a_2 (t - t_0) = \tan\theta_2 t - \tan\theta_2\, t_0$
$\upsilon_r = \upsilon_1 - \upsilon_2 = (\tan\theta_1 - \tan\theta_2)\, t - t_0 (\tan\theta_1 - \tan\theta_2)$
So v_r contineously increases.

A-5. Initial relative velocity
$u_r = 50 - (-50) = 100$
$a_r = 20 - (20) = 0$
$s_r = \upsilon_r t + 1/2\, a_r t^2$
$100 = 100\, t \qquad t = 1$ hr
$s_A = 50\,(1) + 1/2\,(20)\,(1)^2 = 60$ km.

A-6.

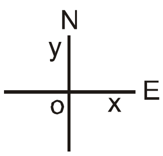


$\vec{V}_A = -500\,\hat{i} \quad \Rightarrow \quad \vec{V}_{GA} + \vec{V}_A = V_{GAS}$



$\vec{V}_{GA} = 1500\,\hat{i} \quad \Rightarrow \quad 1500\,\hat{i} - 500\,\hat{i} = 1000\,\hat{i}$

B-1.



$\vec{V}_r = 50(-\hat{j}) - 50\hat{i} = 50(-\hat{i} - \hat{j})$
i.e., in south west

B-2. $\vec{V}_{12} = \vec{V}_1 - \vec{V}_2$

$|\vec{V}_{12}| = \sqrt{V_1^2 + V_2^2 - 2V_1V_2\cos\theta}$

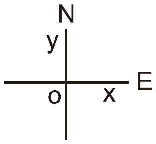
If $\cos\theta = -1$

$|\vec{V}_{12}|_{max} = \sqrt{V_1^2 + V_2^2 + 2V_1V_2}$

$|\vec{V}_{12}|_{max} = (V_1 + V_2)$

So $|\vec{V}_{12}|$ is maximum when $\cos\theta = -1$ and $\theta = \pi$

B-3.



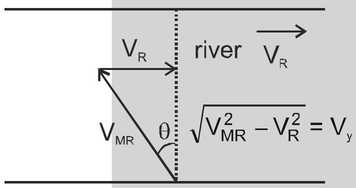
$\vec{V}_1 = 10\hat{i}$

$\vec{V}_2 = v\sin 30\hat{i} + v\cos 30\hat{j} = \frac{\upsilon}{2}\hat{i} + \frac{\upsilon\sqrt{3}}{2}\hat{j}$

$\vec{V}_2 - \vec{V}_1 = \left(\frac{\upsilon}{2} - 10\right)\hat{i} + \frac{\upsilon\sqrt{3}}{2}\hat{j} = \frac{\upsilon\sqrt{3}}{2}\hat{j}$

$\therefore \quad \upsilon/2 - 10 = 0 \quad$ or $\quad \upsilon = 20$

C-1. 15 min = 1/4 hr.



$t = \frac{d}{V_y} \Rightarrow \quad \frac{1}{4} = \frac{1}{\sqrt{V_{MR}^2 - V_R^2}} = \frac{1}{4} = \frac{1}{\sqrt{5^2 - V_R^2}}$

$\Rightarrow \quad V_R = 3$ km/h

C-2. $V_{boat,\ river} = 9$ km/hr.

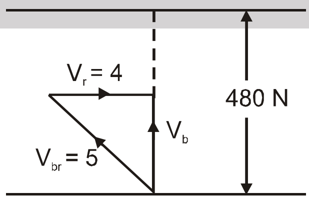
$V_{river,\ ground} = 12$ km/hr.

$V_{boat,\ ground} = (12\hat{i} + 9\hat{j})$ km/hr

$V_{boat,\ ground} = \sqrt{12^2 + 9^2} = 15$ km/hr.

C-3. $V_b = \sqrt{5^2 - 4^2} = 3$ m/s

$t = \frac{480}{3} = 160$ s



C-4. Let velocity of wind be

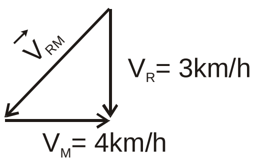
$(240 + v_1)\ 1/2 = 150 \quad \Rightarrow \quad v_1 = 60$

and $v_2 \times 1/2 = 40 \quad \Rightarrow \quad v_2 = 80$

so $v_{air} = \sqrt{v_1^2 + v_2^2} = 100$ km/hr

$\tan\theta = \frac{v_1}{v_2} \qquad \theta = 37^o$ west of south

D-1.



$V_{RH} = \sqrt{V_R^2 + V_M^2} = \sqrt{3^2 + 4^2} = 5$ km/h **Ans.**

D-2. $\vec{V}_r = \upsilon_y \hat{j}$

$\vec{\upsilon}_m = 5\hat{i}$

$\vec{V}_r - \vec{V}_m = (-5)\hat{i} + \upsilon_y \hat{j}$

tan θ = 1 = $\upsilon_y/5$

so υ_y = 5 km/hr

D-3. $\vec{V}_r = 10\hat{j}$

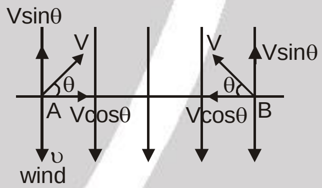
$\vec{V}_c = \upsilon\hat{i}$

$\vec{V}_r - \vec{V}_c = 10\hat{j} - \upsilon\hat{i}$

$|\vec{V}_r - \vec{V}_c| = \sqrt{10^2 + \upsilon^2} = 20$

$\upsilon = 10\sqrt{3}$

D-4. For no drift :



V sinθ = υ

$\sin\theta = \frac{\upsilon}{V}$ $\qquad\therefore\qquad t = t_{AB} + t_{BA}$

$t = \frac{2\ell}{V\cos\theta} = \frac{2\ell}{V\sqrt{1-\frac{\upsilon^2}{V^2}}} \qquad \Rightarrow \qquad t = \frac{2\ell}{\sqrt{V^2 - \upsilon^2}}$

E-1. A and B collide if $\frac{\vec{r}_B - \vec{r}_A}{|\vec{r}_B - \vec{r}_A|} = \pm\frac{\vec{V}_B - \vec{V}_A}{|\vec{V}_B - \vec{V}_A|}$

$= \frac{4\hat{i} + 4\hat{j}}{4\sqrt{2}} = \pm\frac{(x-3)\hat{i} - 4\hat{j}}{|[(x-3)^2 + 4^2]^{1/2}|}$

By comparison

$x - 3 = -4 \qquad \Rightarrow \qquad x = -1.$

ALTERNATE:

$\vec{r}_{AB} = -4\hat{i} - 4\hat{j}$

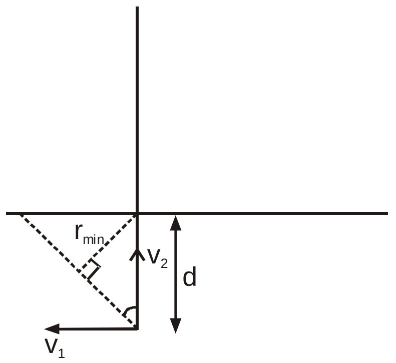
$\vec{v}_{AB} = (3 - x)\hat{i} + 4\hat{j}$

to collide angle between $\vec{r}_{AB}$ & $\vec{v}_{AB}$ should be π i.e.

$= \frac{-4}{3-x} = \frac{-4}{4} - 1 \qquad \Rightarrow \qquad x = -1$

E-2.

Velocity of B w.r.t. A



$\tan\theta = v_1/v_2$

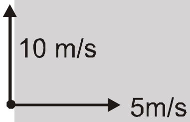
$r_{min} = d \sin\theta = d \cdot \frac{v_1}{\sqrt{v_1^2 + v_2^2}}$.

PART - III

1. In all cases, angle between velocity and net force (in the frame of observer) is in between 0° and 180° (excluding both values, in that path is straight line).

2. (A) $V_{BA} = 10 + 10 = 20$
so distance b/w B and A in 2sec. = 2 × 20 = 40 m

(B)

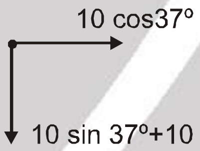


$\vec{V}_{BA} = 5\hat{i} - 10\hat{j}$

$\Rightarrow$ $|V_{BA}| = \sqrt{25+100} = 5\sqrt{5}$

Distance between A and B in 2 sec. = $10\sqrt{5}$ m

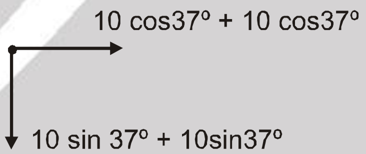
(C) $\vec{V}_{BA}$:-



so $|\vec{V}_{BA}| = 8\sqrt{5}$

so distance between A and B in 2 sec. = 2 × $8\sqrt{5}$ = $16\sqrt{5}$

(D) $\vec{V}_{BA}$: -



so $|\vec{V}_{BA}| = 20$

so distance between A and B in 2 sec. = 2 × 20 = 40 m.

EXERCISE # 2

PART - I

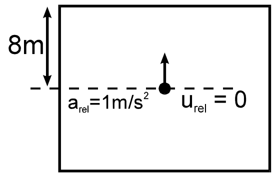
1. Relative velocity between either car (1st or 2nd) and 3rd car = u + 30
where u = velocity of 3rd car
Relative Displacement = 5 km
Time interval = 4 min.

$\therefore$ $u + 30 = \frac{5}{4}$ km/min $= \frac{5 \times 60}{4}$ km/h = 75 $\Rightarrow$ u = 45 km/h

2. $V_{rel} = \frac{S_{rel}}{t} = \frac{1000}{100} = 10$ m/s.

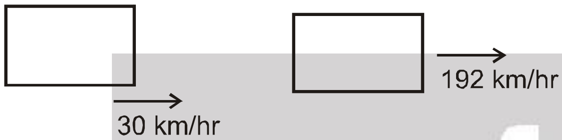
$\therefore \quad V_S - V_B = 10 \quad \Rightarrow \quad V_S = 10 + V_B = 10 + 10 = 20$ m/s. **Ans.**

3. Relative to lift initial velocity and acceleration of coin are 0 m/s and 1 m/s^2 upwards



$\therefore \quad 8 = \frac{1}{2}(1)\ t^2 \quad$ or $\quad t = 4$ second

4.



30 km/h = $\frac{25}{3}$ m/s, 192 km/hr = $\frac{160}{3}$ m/s

Muzzle speed = velocity of bullet w.r.t. revolver

= velocity of bullet w.r.t. van

$150 = V_b - V_v$

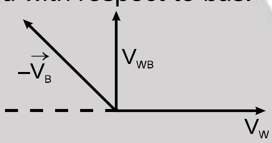
$150 = V_b - \frac{25}{3} \quad \Rightarrow \quad V_b = \frac{475}{3}$ m/s w.r.t. ground

Now speed with which bullet hit thief's car

= velocity of bullet w.r.t car = $V_{bc} = V_b - V_c$

$= \frac{475}{3} - \frac{160}{3} = \frac{315}{3} = 105$ m/s **Ans.**

5. Flag will flutter in the direction of wind with respect to bus.



and $\vec{V}_{WB} = \vec{V}_W - \vec{V}_B = \vec{V}_W + (-\vec{V}_B)$ (Addition of two vector always lies between them)

$(-\vec{V}_B)$ must lie in any direction between north & west. So bus will be moving in any direction between south east. **(C)**

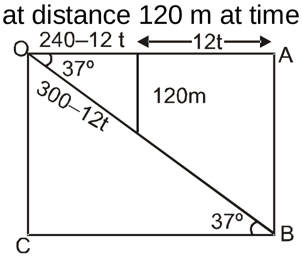
6. Let $\hat{i}$ and $\hat{j}$ be unit vectors in direction of east and north respectively.

$\therefore \quad \vec{V}_{DC} = 20\hat{j}$, $\vec{V}_{BC} = 20\hat{i}$ and $\vec{V}_{BA} = -20\hat{j}$

$\therefore \quad \vec{V}_{DA} = \vec{V}_{DC} + \vec{V}_{CB} + \vec{V}_{BA} = 20\hat{j} - 20\hat{i} - 20\hat{j} = -20\hat{i}$

$\therefore \quad \vec{V}_{DA} = -20\hat{i}$

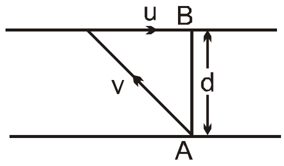
7. Position of P and Q when they are at distance 120 m at time t after motion start



Velocity of Q along y-direction is initially 12cos37°.

Later on it increases it to 15cos37º = 12 m/s
Earlier Q was travelling with less velocity along y direction. So, it will reach point O later.
So P reaches first at point O

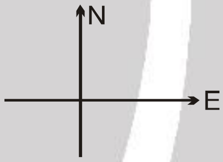
8. Let v = man's speed in still water and
u = speed of river water

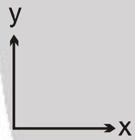


$\Rightarrow \quad t = \dfrac{d}{\sqrt{v^2 - u^2}} \quad \Rightarrow \quad v^2 - u^2 = \dfrac{d^2}{t^2}$

$\Rightarrow \quad T = \dfrac{d}{v+u} \quad \Rightarrow \quad (v+u)^2 = \dfrac{d^2}{T^2} \quad \Rightarrow \quad \dfrac{(v+u)^2}{v^2-u^2} = \dfrac{t^2}{T^2}$

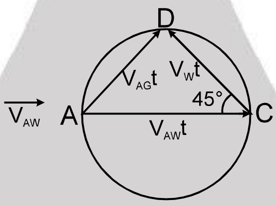
$\Rightarrow \quad \dfrac{v+u}{v-u} = \dfrac{t^2}{T^2}$





$\Rightarrow \quad \dfrac{(v+u)+(v-u)}{(v+u)-(v-u)} = \dfrac{t^2+T^2}{t^2-T^2} \quad \Rightarrow \quad \dfrac{v}{u} = \dfrac{t^2+T^2}{t^2-T^2}$

9. In absence of wind A reaches to C and in presence of wind it reaches to D in same time so wind must deflect from C to D so wind blow in the direction of CD

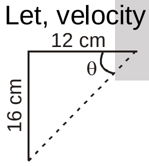


$\vec{V}_{AG} = \vec{V}_{AW} + \vec{V}_{WG} \quad \Rightarrow \quad \vec{V}_{AG} \ t = \vec{V}_{AW} t + \vec{V}_{WG} \ t$

AC = $\vec{V}_{AW} t$

CD = $\vec{V}_{WG} t$

10. $V_{R/G(x)} = 0$, $V_{R/G(y)} = 10$ m/s
Let, velocity of man = v



$\tan\theta = \dfrac{16}{12} = \dfrac{4}{3}$

Then, $v_{R/man}$ = v (opposite to man)
For the required condition :

$\tan\theta = \dfrac{V_{R/M(y)}}{V_{R/M(x)}} = \dfrac{10}{v} = \dfrac{4}{3} \quad \Rightarrow \quad V = \dfrac{10 \times 3}{4} = 7.5$ **Ans.**

11. $T = \frac{2(5)}{10} = 1sec$

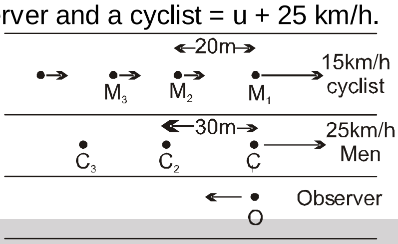
h = 50 + 3(1) = 53m

PART - II

1. Let u = speed of observer.

Relative velocity between observer and a man = u + 15 km/h.

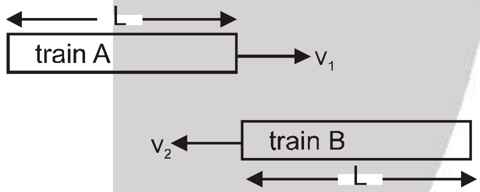
Relative velocity between observer and a cyclist = u + 25 km/h.



Hence, to for a man and a cyclist to meet simultaneously

$$\frac{20m}{(u+15)\,km/h} = \frac{30m}{(u+25)\,km/h} \Rightarrow u = 5 \text{ km/h}$$

2.



$t_1 = 3 = \frac{2L}{v_1 + v_2} \Rightarrow \quad v_1 + v_2 = \frac{2L}{3}$ ....(i)

$t_2 = 2.5 = \frac{2L}{1.5v_1 + v_2} \quad 1.5\,v_1 + v_2 = \frac{4L}{5}$ ....(ii)

by (i) and (ii)

$v_1 = \frac{4L}{15}$; $v_2 = \frac{2L}{5}$

Now, $t_3 = \frac{2L}{|v_1 - v_2|} = \frac{2L}{2L/15} = 15$ sec.

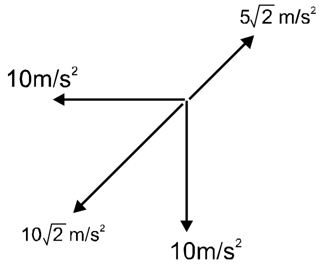
3. B catches C in time t then $t = \frac{d}{u-10}$

Seperation by this time has increased by 'd' between A and C hence

$(10 - 5) \times \frac{d}{(u-10)} = d$

u = 15 m/s

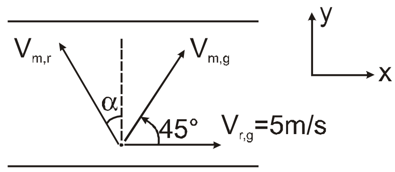
4.



w.r.t. man

5. $\vec{V}_{m,g} = \vec{V}_{m,r} + \vec{V}_{r,g}$

As resulting velocity $\vec{V}_{m,g}$ is at 45° with river flow



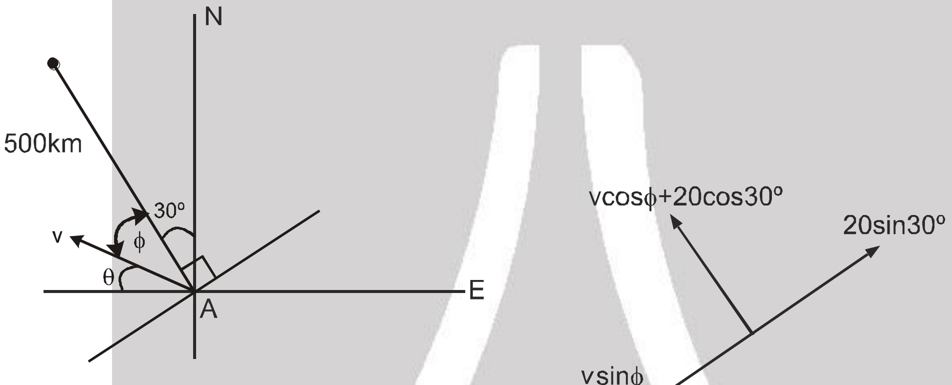
i.e. $V_{r,g} - V_{m,r} \sin\alpha = V_{m,g} \cos 45°$ ....(1)

and $\dfrac{60\text{m}}{V_{mr}\cos\alpha} = 6$ sec. ....(2)

Solving (1) & (2)

$V_{m,r} = 5\sqrt{5}$ m/s

6.



Velocity of plane w.r.t. ground

Velocity of plane w.r.t ground is along AB so perpendicular component (to line AB) of velocity is zero.

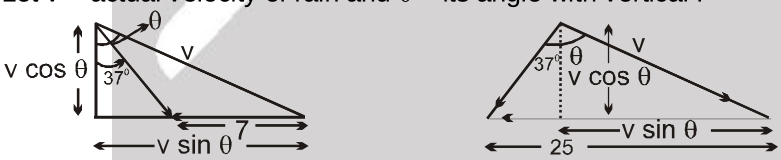
$v \sin\phi = 20 \sin 30°$

$v \sin\phi = 20 \sin 30°$

$\sin\phi = \dfrac{10}{150} = \dfrac{1}{15}$

$\phi = \sin^{-1}\left(\dfrac{1}{15}\right)$

7. Let v = actual velocity of rain and θ = its angle with vertical :



In fig. (A)

$v \sin\theta = 7 + v \cos\theta . \tan 37° = 7 + 3/4\, v \cos\theta$

$\Rightarrow$ $4 v \sin\theta - 3 v \cos\theta = 28$ ....(1)

In fig. (B)

$25 = v \sin\theta + v \cos\theta . \tan 37° = v \sin\theta + 3/4\, v \cos\theta$

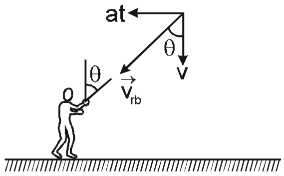
$\Rightarrow$ $4 v \sin\theta + 3 v \cos\theta = 100$ ....(2)

Solving (1) and (2)

v = 20m/s and θ = 53°

8. At any time t, rain will appear to the boy as shown in picture.

$\tan\theta = at/v$



$\vec{v}_{rb}$ = velocity of rain w.r.t. boy

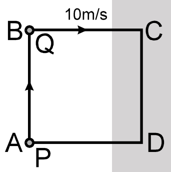
Boy should hold his umbrella at an angle θ from the vertical

$\therefore \quad \tan\theta = \frac{at}{v} \qquad \sec^2\theta \frac{d\theta}{dt} = \frac{a}{v}$

$$\Rightarrow \quad \frac{d\theta}{dt} = \frac{a}{v\sec^2\theta} = \frac{a}{v[1+\tan^2\theta]} = \frac{a}{v\left[1+\frac{a^2t^2}{v^2}\right]} = \frac{av}{v^2+a^2t^2} = \frac{2\times 2}{4+4t^2} = \frac{1}{1+t^2}$$

$\frac{d\theta}{dt} = \frac{1}{1+t^2}$ **Ans.** $\frac{d\theta}{dt}$ **=** $\frac{1}{1+t^2}$

9.



a = side of square = 8m

They meet when Q displace 8 × 3 m more than P

$\Rightarrow$ Relative displacement = Relative velocity × time.

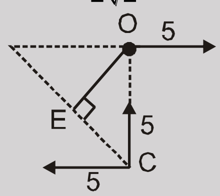
8 × 3 = (10 – 2) t $\Rightarrow$ t = 3 sec **Ans. 3 sec**

10. The positions of the persons at 12:00 PM will be as shown in figure. Such that OC = 5 × $\frac{3}{2}$ km = $\frac{15}{2}$ km

Velocity of man at C with respect to man at O will be along CE such that tanθ = 5/5 = 1

$\therefore \quad \theta = 45°$

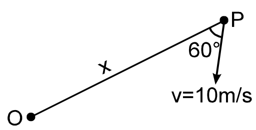
$\therefore$ Least distance = OE = OC sin 45° = $\frac{15}{2\sqrt{2}}$ km **Ans.**



Time taken = $\frac{CE}{5\sqrt{2}} = \frac{15}{2\sqrt{2}\times 5\sqrt{2}} = \frac{3}{4}$ hr.

So, the person will be closest at 12:45 PM **Ans.**

11.



Velocity of approach of P and O is → $\frac{dx}{dt}$ = v cos 60° = 5 m/s

It can be seen that velocity of approach is always constant.

$\therefore$ P reaches O after = $\frac{100}{5}$ = 20 sec.

PART - III

1. Relative Intial velocities

$u_r = 20 - (0) = 20$ m/s

Relative acceleration

$a_r = 0$

Relative velocity between them after time

$v_r = u_r + a_r .t$

= 20m/s

= constant

⇒ (A) is correct

⇒ Since they are thrown from same height

⇒ Speed is same after reaching ground

⇒ Same KE when they hit the ground

⇒ (B) is correct

The time taken by the first stone to come to same height from where it was thrown.

$$\frac{2u}{g} = \frac{2 \times 20}{10}$$

∴ Time interval between two stone when both are at A and going downwards = 4 – 2 = 2s.

Since, relative velocity is Constant between them. So time interval between their hitting the ground = 2 s.

⇒ (C) is correct

Option (D) is obvious from conservation of energy.

2. For first case (when lift is ascending with an acceleration a)

$$t_1 = \frac{2v}{g+a} \qquad \text{...........(i)}$$

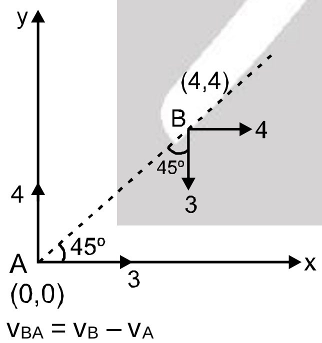
for second case (when lift is descending with an acceleration a)

$$t_2 = \frac{2v}{g-a} \qquad \text{...........(ii)}$$

on solving equation (i) and (ii) we get

$$V = \frac{g t_1 t_2}{t_1 + t_2} \quad \& \quad a = g\left(\frac{t_2 - t_1}{t_1 + t_2}\right)$$

3.



$v_{BA} = v_B - v_A$

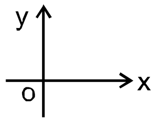
$= [4\hat{i} - 3\hat{j}] - [3\hat{i} + 4\hat{j}] = \hat{i} - 7\hat{j}$

v_{app} = 4 cos45º + 3 cos45º + 3 cos45º – 4 cos45º

= 6 cos45º

= $3\sqrt{2}$ m/s

4. u_T = 15 m/s (velocity of truck)



Range = 60 m

Range = distance travelled by truck $u_T \times T$

60 = 15 × T ⇒ T = 4s = Time of flight (of ball)

$T = \frac{2u_y}{g}$ where u_y = Vertical component of ball's vel. {wrt ground}

$\therefore \quad \frac{T \times g}{2} = u_y$ i.e, $u_y = \frac{4 \times 10}{2}$ = 20 m/s

Now vel. of truck = u_x = horizontal component of ball's vel. (wrt ground)

⇒ u_x = 15 m/s (wrt ground)

This is because both cover same horizontal distance in same time with constant velocity along horizontal.

Now, velocity, of vall wrt track = V_{BT} Then $V_{BTx} = V_{Bx} - V_{Tx}$

i.e., velocity ball wrt truck (along x axis) = velocity of ball (wrt earth, along x axis) – velocity of truck (along x axis)

$\therefore \quad V_{BT\,x}$ = 15 – 15 = 0

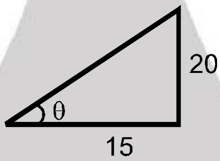
Similarly, $V_{BT\,y} = V_{By} - V_{Ty}$ = 20 – 0

V_{BTy} = 20 m/s.

⇒ $\vec{V}_{BT} = \vec{V}_{BTx} + \vec{V}_{BTy} = 0 + 20 \text{ m/s } \hat{j}$

⇒ velocity of ball wrt truck = 20 m/s upwards

velocity of ball, $\vec{V} = \vec{V}_x + \vec{V}_y \qquad \vec{V} = 15\hat{i} + 20\hat{j}$

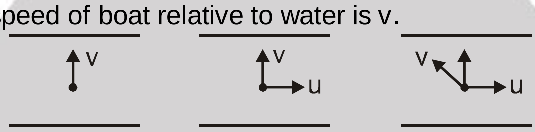


tanθ = 4/3 θ = 53º

$\therefore$ Speed = $|\vec{V}| = \sqrt{15^2 + 20^2} = 5\sqrt{3^2 + 4^2}$ = 25 m/s

i.e., vel. of ball (wrt ground) = 25 m/s at an angle of 53º with the horizontal (as shown)

5. Speed of river is u and speed of boat relative to water is v.



Boat A river frame Boat A ground frame Boat B ground frame

Speed of boat A observed from ground = $\sqrt{u^2 + v^2}$

Speed of boat B observed from ground = $\sqrt{v^2 - u^2}$

From river frame, speed of boat A and B will be same.

6. (A) Absolute velocity of ball = 40 m/s (upwards)

$h_{max} = h_i = f_f = 10 + \frac{(40)^2}{2 \times 10}$

h = 90 m

(B) Maximum height from lift = $\frac{(30)^2}{2 \times 10}$ = 45 m

(C) The ball unless meet the elevator again when displacement of ball = displacement of lift

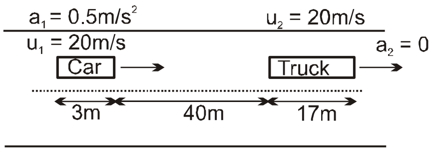
40 t – 1/2 × 10 × t^2 = 10 × t ⇒ t = 6s.

(D) with respect to elevator V_{ball} = 30 m/s downward $\therefore$ V_{ball} with respect to ground = 30 – 10 = 20 m/s

PART - IV

1. Displacement of car relative to truck

$x_r = 40 + 17 + 3 + 40 = 100$ m.

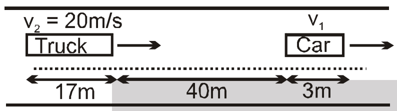


t = 0

Initilly

Relative initial velocity between car and truck

$u_r = 20 - 20 = 0$



t = t

Finally

Relative acceleration between car and truck

$a_r = 0.5 - 0 = 0.5$ m/s^2

Let required time = t .

$\therefore$ II equation of motion

$x_r = u_r.t + 1/2\, a_r.t^2$

$\Rightarrow$ $100 = 0 + 1/2 \times 0.5 \times t^2$

$\Rightarrow$ t = 20 sec.

2. Distance travelled by car

$x_c = ut + 1/2\, at^2$

$= 20 \times 20 + 1/2 \times 0.5 \times 20^2 = 500$m

3. Final speed of the car

$= u + at$

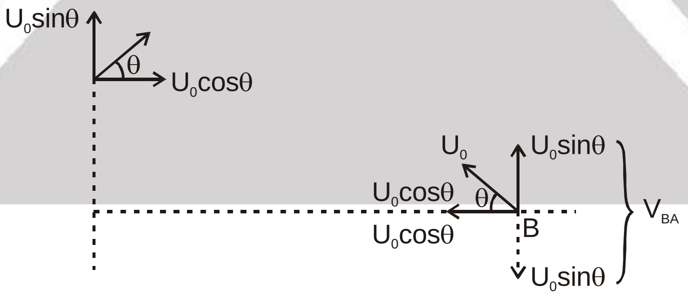
$= 20 + 0.5 \times 20 = 30$m/s.

4 & 5. The path of a projectile as observed by other projectile is a straight line.

$V_A = u\cos\theta\,\hat{i} + (u\sin\theta - gt)\,\hat{j}$. $V_{AB} = (2u\cos\theta)\,\hat{i}$

$V_B = -u\cos\theta\,\hat{i} + (u\sin\theta - gt)\,\hat{j}$.

$a_{BA} = g - g = 0$



The vertical component $u_0 \sin\theta$ will get cancelled. The relative velocity will only be horizontal which is equal to $2u_0 \cos\theta$.

Hence, B will travel horizontally towards left with respect to A with constant speed $2u_0 \cos\theta$ and minimum distance will be h.

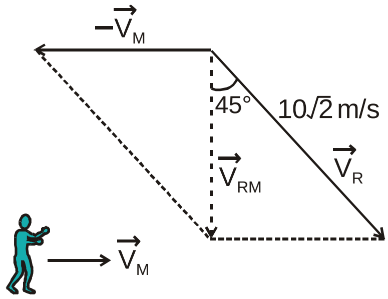
6. Time to attain this separation will be $\frac{S_{rel}}{V_{rel}} = \frac{\ell}{2u_0\cos\theta}$.

7 to 9. In the first case :

From the figure it is clear that

$\vec{V}_{RM}$ is 10 m/s downwards and

$\vec{V}_M$ is 10 m/s towards right.



In the second case :

Velocity of rain as observed by man becomes times in magnitude.

∴ New velocity of rain

$\vec{V}_{R'} = \vec{V}_{R'M} + \vec{V}_M$

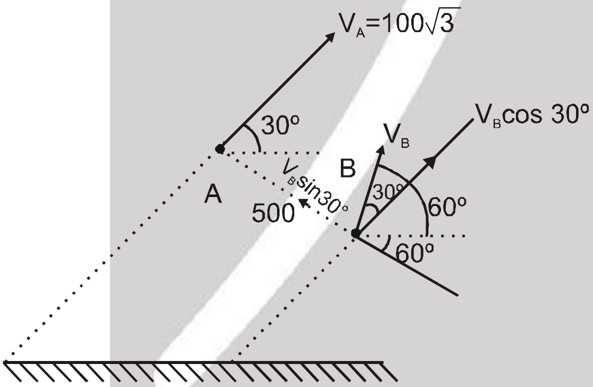
∴ The angle rain makes with vertical is $\tan\theta = \frac{10}{10\sqrt{3}}$

or $\theta = 30°$

∴ Change in angle of rain = 45 – 30 = 15°.

EXERCISE # 3
PART - I

1.



For relative motion perpendicular to line of motion of A

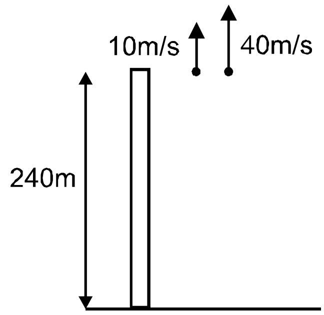
$V_A = 100\sqrt{3} = V_B \cos 30°$

$\Rightarrow$ $V_B = 100$ m/s

$t_0 = \frac{50}{V_B \sin 30°} = \frac{500}{200 \times \frac{1}{2}} = 5\,\text{sec}$ **Ans.**

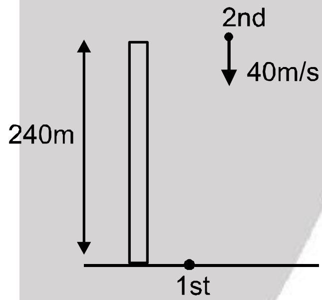
PART - II

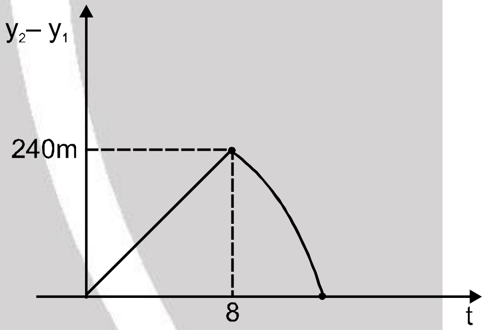
1.



$-240 = 10\,t - 1/2 \times 10t^2$
$5t^2 - 10t - 240 = 0$
$t^2 - 2t - 48 = 0$
$t^2 - 8t + 6t - 48 = 0$
$t = 8, -6$

The first particle will strike ground at 8 seconds upto 8 second, relative velocity is 30 m/s and relative acceleration is zero. After 8 second magnitude of relative velocity will increase upto 12 seconds when second particle strikes the ground.





2. $\vec{V}_r = 40\hat{i} + 50\hat{j}$

$\vec{r}_r = -80\hat{i} - 150\hat{j}$

$t_{min} = \dfrac{|\vec{V}_r \cdot \vec{r}_r|}{|\vec{V}_r|^2} = \dfrac{10700}{4100} = \dfrac{107}{41}$ = 2.6 sec.

3. $X_1 = -3t^2 + 8t + 10$

$\vec{v}_1 = (-6t + 8)\hat{i} = 2\hat{i}$

$Y_2 = 5 - 8t^3$

$\vec{v}_2 = -24t^2\hat{j}$

$\sqrt{v} = |\vec{v}_2 - \vec{v}_1| = |-24\hat{j} - 2\hat{i}|$

$\sqrt{v} = \sqrt{24^2 + 2^2}$

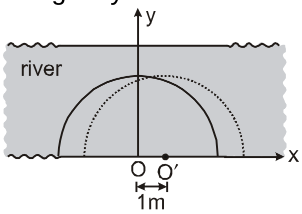
$v = 580$

HIGH LEVEL PROBLEMS (HLP)

1. **Method - 1**

If the river is still, the man will be at a distance 3 meters from origin O after 1 second. The locus of all the point where man can reach at t = 1 second is a semicircle of radius 3 and centre at O (dotted semicircle shown in figure).

The river flows to right with a speed 1 m/s. Hence there shall be additional shift in position by 1 m/s × 1 sec = 1m towards right. Hence the locus of all points giving possible position after one second will be the dotted semicircle shifted to right by 1 m as shown in figure.



Hence locus all the points where the man can be at t = 1 sec. is a semicircle of radius 3 and centre at O′ (1 m, 0 m)

$\therefore$ Equation of locus of all the points is

$(x-1)^2 + (y-0)^2 = 3^2$

or $(x-1)^2 + y^2 = 9$

Method - 2

Let the relative velocity of the man make angle ‘θ’ with the x-axis.

Then at time ‘t’ :

$x = (3\cos\theta + 1)\,t$

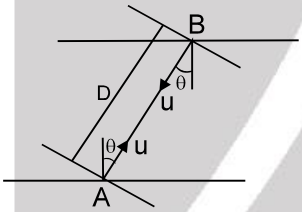
and $y = 3\sin\theta\, t$

$\Rightarrow$ $(x-t)^2 + y^2 = (3\cos\theta)^2 t^2 + (3\sin\theta)^2 t^2$

$(x-t)^2 + y^2 = 9t^2$

at $t = 1$ sec. the required equation is $(x-1)^2 + y^2 = 9$.

2. (a)

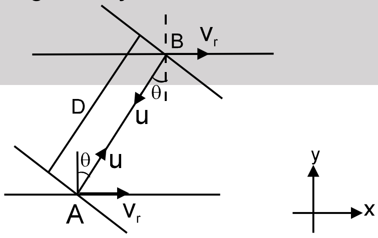


Velocity of approach along line joing them

$= u - V_r \sin\theta + u + V_r \sin\theta = 2u$

So time $t = \dfrac{D}{2u}$ **Ans.**

(b) For path to be at right angle to each other, their velocity vector with respect to ground must be right angle. Taking axis system as shown



$\vec{V}_A = (u\sin\theta + v_r)\,\hat{i} + u\cos\theta\,\hat{j}$

$\vec{V}_B = (v_r - u\sin\theta)\,\hat{i} - u\cos\theta\,\hat{j}$

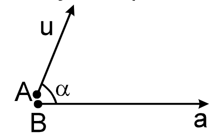
For $\vec{V}_A$ & $\vec{V}_B$ to perpendicular $\vec{V}_A \cdot \vec{V}_B = 0$

$(u\sin\theta + v_r)(v_r - u\sin\theta) - u^2\cos^2\theta = 0$

$\Rightarrow$ $v_r = u$

Speed of river should be equal to the speed of the swimmer relative to river. Ans.

3. Let particle A is moving with uniform velocity and particle B is moving with constant acceleration.



At any time, velocity of B with respect to A is

$\vec{V}_{BA} = \vec{V}_{BG} - \vec{V}_{AG} = (\vec{a})t - \vec{u}$

$V_{BA} = \left| \vec{V}_{BA} \right| = \sqrt{u^2 + a^2t^2 - 2uat\cos\alpha}$

For relative velocity to be least

$$\frac{dv_{BA}}{dt} = 0 \quad \Rightarrow \quad 2a^2t - 2ua\cos\alpha = 0 \quad \Rightarrow \quad t = \frac{u\cos\alpha}{a}$$

$(V_{BA})_{least} = \sqrt{u^2 + u^2\cos^2\alpha - 2u^2\cos^2\alpha}$

$(V_{BA})_{least} = u \sin\alpha$

At time $t = \dfrac{u\cos\alpha}{a}$

distance between the two particles

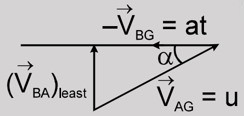
$$s_{BA} = \sqrt{x^2 + y^2} = \sqrt{\left(-u\cos\alpha t + \frac{1}{2}at^2\right)^2 + (u\sin\alpha t)^2}$$

$$= \sqrt{\left(-\frac{u^2\cos^2\alpha}{a} + \frac{u^2\cos^2\alpha}{2a}\right)^2 + \left(\frac{u^2\sin\alpha\cos\alpha}{a}\right)^2}$$

$$= \sqrt{\frac{u^4\cos^4\alpha}{4a^2} + \frac{u^4\sin^2\alpha\ \ \cos^2\alpha}{a^2}} = \frac{u^2\cos\alpha}{2a}\sqrt{\cos^2\alpha + 4\sin^2\alpha}$$

$$s_{BA} = \frac{u^2\cos\alpha}{2a}\sqrt{1 + 3\sin^2\alpha}$$

Alternative Method :



$(V_{BA})_{least} = u \sin\alpha$

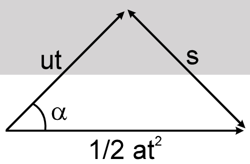
relative velocity of B w.r.t. A is least at time t which is given by

$u\cos\alpha = at$

$t = \dfrac{u\cos\alpha}{a}$

for distance between A and B at the time t

By cosine formula

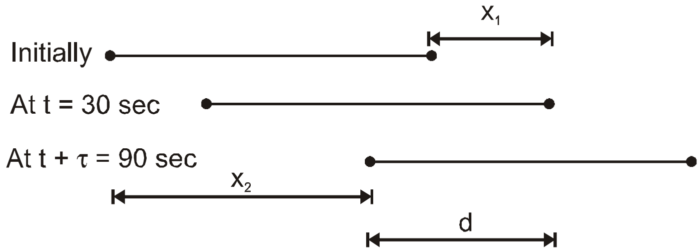


$$s = \sqrt{u^2t^2 + \left(\frac{1}{2}at^2\right)^2 - 2(ut)\left(\frac{1}{2}at^2\right)\cos\alpha}$$

$= \dfrac{u^2\cos\alpha}{2a}\sqrt{1 + 3\sin^2\alpha}$ using $t = \dfrac{u\cos\alpha}{a}$

Ans. (a) $(v_{rel})_{least} = u \sin\alpha$ (b) separation = $\dfrac{u^2\cos\alpha}{2a}\sqrt{1 + 3\sin^2\alpha}$

4. In the frame of train, the distance between A and B remains constant which is equal to ℓ = 350. Hence, in the frame of train the distance between two events is equal to AB = ℓ = 350 m.



Distance between these points with respect to ground

$d = \ell + x_1 - x_2$

$= \ell + \frac{1}{2}\omega t^2 - \frac{1}{2}\omega(t+\tau)^2$

$= \ell - \omega\tau\left(t + \frac{\tau}{2}\right) \simeq 240$ m

Time between these two events = τ = 60 sec.

Velocity of frame V = $\frac{240}{60}$ = 4 m/s.

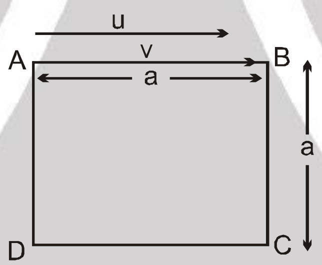
5. (a) Relative velocity along AB $\rightarrow$ u + v

BC $\rightarrow \sqrt{v^2 - u^2}$

CD $\rightarrow$ v – u

DA $\rightarrow \sqrt{v^2 - u^2}$

$\therefore$ time taken



$t = \frac{a}{v+u} + \frac{a}{\sqrt{v^2-u^2}} + \frac{a}{v-u} + \frac{a}{\sqrt{v^2-u^2}}$

$= \frac{2a}{\sqrt{v^2-u^2}} + a\left(\frac{1}{v+u} + \frac{1}{v-u}\right)$

$= \frac{2a}{\sqrt{v^2-u^2}} + a\left(\frac{v+u+v-u}{v^2-u^2}\right)$

$= \frac{2a}{\sqrt{v^2-u^2}} + \frac{2av}{v^2-u^2}$

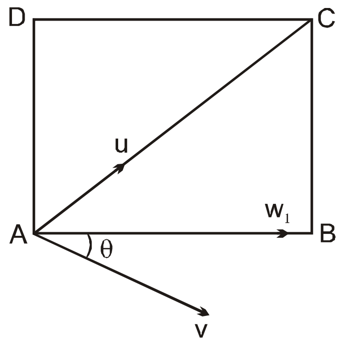
$= 2a\left[\frac{\sqrt{v^2-u^2}+v}{v^2-u^2}\right]$

$t = 2a\left[\frac{v+\sqrt{v^2-u^2}}{v^2-u^2}\right]$

$t = \frac{2a}{v^2-u^2}\left[v + \sqrt{v^2-u^2}\right]$

(b) **Along AB**

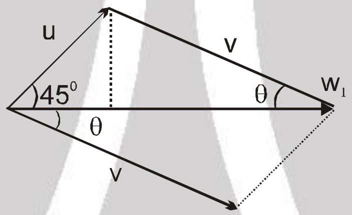
Let w_1 is the resultant velocity



$$w_1 = \frac{u}{\sqrt{2}} + v\cos\theta$$

$$= \frac{u}{\sqrt{2}} + v\sqrt{1-\sin^2\theta} = \frac{u}{\sqrt{2}} + v\sqrt{1-\frac{u^2}{2v^2}}$$

Similarly, we can show that



Along BC Resultant velocity

$$w_2 = v\sqrt{1-\frac{u^2}{2v^2}} - \frac{u}{\sqrt{2}}$$

Along CD Resultant velocity

$$w_3 = v\sqrt{1-\frac{u^2}{2v^2}} - \frac{u}{\sqrt{2}}$$

Along DA Resultant velocity

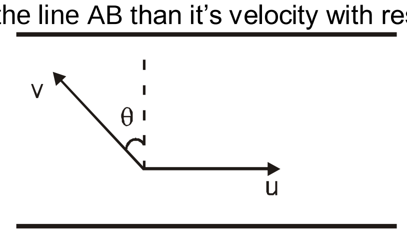
$$w_4 = \frac{u}{\sqrt{2}} + v\sqrt{1-\frac{u^2}{2v^2}}$$

∴ Total time taken :-

$$t = \frac{a}{w_1} + \frac{a}{w_2} + \frac{a}{w_3} + \frac{a}{w_4}$$

$$= \frac{2a}{\frac{u}{\sqrt{2}} + v\sqrt{1-\frac{u^2}{2v^2}}} + \frac{2a}{v\sqrt{1-\frac{u^2}{2v^2}} - \frac{u}{\sqrt{2}}} = \frac{2\sqrt{2}a\sqrt{2v^2-u^2}}{v^2-u^2}$$

6. Let man swim at angle θ with the line AB than it's velocity with respect to ground any time t is as shown.



along y-axis

$$V_y = V\cos\theta = 5\cos\theta$$

So time taken to cross the river $t = \frac{d}{5\cos\theta}$ ....(i)

and velocity along x-axis

$$V_x = u - V\sin\theta \quad \Rightarrow \quad \frac{dx}{dt} = \frac{t}{2} - 5\sin\theta$$

$$\int_0^x dx = \int_0^t \left(\frac{t}{2} - 5\sin\theta\right)dt$$

$$x = \frac{t^2}{4} - 5t\sin\theta$$

For complete motion x = 0

$$\Rightarrow \quad \frac{t^2}{4} - 5t\sin\theta = 0 \quad \Rightarrow \quad t = 20\sin\theta \quad \text{....(ii)}$$

by eq (i) and (ii)

$$100\cos\theta\sin\theta = d = 48 \text{ m}$$

$$\sin 2\theta = \frac{24}{25} \quad \Rightarrow \quad 2\theta = \sin^{-1}\left(\frac{24}{25}\right)$$

$$2\theta = 74^\circ \quad \Rightarrow \quad \theta = 37^\circ$$

$$2\theta = 106^\circ \quad \Rightarrow \quad \theta = 53^\circ$$

from eq (ii) (ii)

for $\theta = 37^\circ \Rightarrow$ t = 12 sec

for $\theta = 53^\circ \Rightarrow$ t = 16 sec

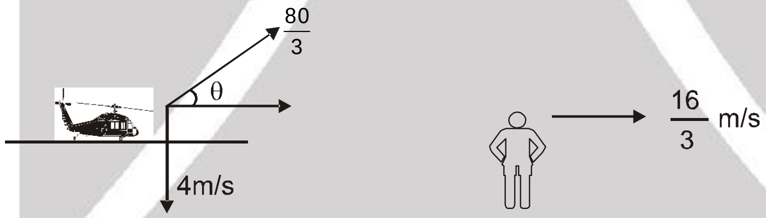
also y-coordinate of man

$$y = 5t\cos\theta \quad \text{....(iii)}$$

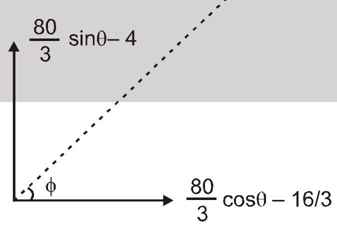
$$x = \frac{y^2}{100\cos^2\theta} - y\tan\theta$$

by eq (ii) and (iii) $\quad x = \frac{y^2}{64} - \frac{3y}{4}$

7. Velocity of helicopter and child is as shown



Now velocity of helicopter with respect to child

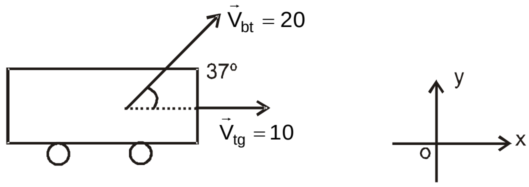


$$\tan\phi = \frac{\frac{80}{3}\sin\theta - 4}{\frac{80}{3}\cos\theta - \frac{16}{3}} = \frac{600}{800} \quad \Rightarrow \quad \tan\theta = \frac{3}{4} \quad \Rightarrow \quad \theta = 37^\circ$$

Now horizontal velocity $= \frac{80}{3}\cos\theta - \frac{16}{3} = \frac{80}{3}\cos 37^\circ - \frac{16}{3} = 16$ m/s

So time taken $= \frac{800}{16} = 50$ sec.

8. t = 0



$\vec{v}_{bg} = \vec{v}_{bt} + \vec{v}_{tg}$

$\vec{v}_{bt} = 20 \cos 37^o \hat{i} + 20 \sin 37^o \hat{j}$

$= 20 \times \frac{4}{5} \hat{i} + 20 \times \frac{3}{5} \hat{j} = 16\hat{i} + 12\hat{j}$

$\vec{v}_{tg} = 10\hat{i}$

$\therefore \quad \vec{v}_{bg} = 26\hat{i} + 12\hat{j}$

At t = 0 $\quad x_b = 0 \quad x_t = 0$

Let they meet after time t

$X_{b(t)} - X_{b(0)} = 26t$

$X_{train(t)} - X_{train(o)} = 10t + \frac{1}{2}at^2$

Let The ball return to train after time t $\quad \therefore \quad t = \frac{2.v_y}{g}$

$t = \frac{2 \times 12}{10} = \frac{12}{5}$ s

$X_{b(t)} = X_{train\,(t)} \quad \therefore \quad 26.t = 10\,t + \frac{1}{2}at^2$

$16 = \frac{1}{2}at$

$\frac{32}{t} = a = \frac{40}{3}$ m/s^2

$V_{train(t)} = 10 + at.$

$V_{train\,(t)} = 10 + \frac{40}{3} \times \frac{12}{5} = 10 + 32 = 42$ m/s

9. Let after time t, A is at P and B is at Q. Let T = Total time. Their velocities after time t

$V_A = at \quad$ ....(1)

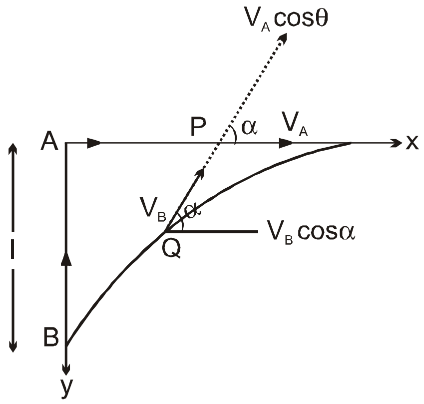
$V_B = bt \quad$ ....(2)

Let distance PQ = x.

Velocity of approch along PQ

$= V_B - V_A \cos\alpha$

$\Rightarrow \quad -\frac{dx}{dt} = V_B - V_A \cos\alpha = bt - at\cos\alpha$



$\Rightarrow \quad -\int_{\ell}^{0} dx = \int_{0}^{T} (bt - at\cos\alpha)dt$

$\Rightarrow \quad \ell = \frac{bT^2}{2} - a\int_{0}^{T} t\cos\alpha\, dt$ .... (3)

For motion along x-axis :

$\int_{0}^{T} V_B \cos\alpha\, dt = \frac{1}{2}aT^2$

$\int_{0}^{T} bt\cos\alpha\, dt = \frac{1}{2}aT^2$

$\Rightarrow \quad \int_{0}^{T} t\cos\alpha\, dt = \frac{1}{2}\frac{aT^2}{b}$ .....(4)

Put into (3) :

$\ell = \frac{bT^2}{2} - a \times \frac{1}{2}\frac{aT^2}{b}$

$\Rightarrow \quad T = \sqrt{\frac{\ell b}{b^2 - a^2}}$

Now, distance travelled by B

$s = \int_{0}^{T} V_B\, dt = \int_{0}^{T} bt\, dt = \frac{1}{2}bT^2$

$\Rightarrow \quad s = \frac{1}{2}b.\frac{2\ell b}{b^2 - a^2} = \frac{\ell b}{b^2 - a^2}$ **Ans.**

10. v = velocity of swimmer in still mater = 2.5 km/hr.
u = velocity of stream = 2 km/hr.
For Swimmer 1
Time taken to reach point B

$t = \frac{d}{\sqrt{v^2 - u^2}}$ .... (1)

For Swimmer 2
at v_0 = velocity of walking along shore
Time to reach C

$t_1 = \frac{d}{v}$

Time taken in coming from C to B

$t_2 = \frac{d\frac{u}{v}}{v_0} = \frac{ud}{v\, v_0}$

$\therefore$ Total time

$t = t_1 + t_2$

$= \frac{d}{v} + \frac{ud}{v\,v_0}$ .... (2)

From (1) and (2)

$\frac{d}{\sqrt{v^2 - u^2}} = \frac{d}{v} + \frac{ud}{v\,v_0}$

$\Rightarrow$ $v_0 = \left(\frac{\sqrt{v^2 - u^2}}{v - \sqrt{v^2 - u^2}}\right) \times u$

Putting u = 2 and v = 2.5 km/hr ; v = 2.52.5 km/hr

$\Rightarrow$ v_0 = 3 km/hr.

11. We have

$\vec{v} = \vec{v}_0 + \vec{v}'$ ...(1)

From the vector diagram [of equation (1)] and using properties of triangle

$v'^2 = v_0^2 + v^2 - 2v_0v\cos(\pi - \phi)$

or, $v' = \sqrt{v_0^2 + v^2 + 2v_0v\cos\phi}$ = 40 km/hr ...(2)

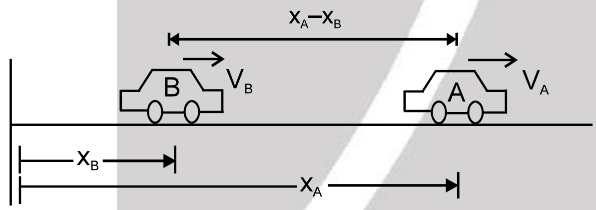
and $\frac{v'}{\sin(\pi - \phi)} = \frac{v}{\sin\theta}$ or, $\sin\theta = \frac{v\sin\phi}{v'}$

or $\theta = \sin^{-1}\left(\frac{v\sin\phi}{v'}\right)$

Using (2) and putting the values of v and d

$\theta = 19°$

12.



As given

$(V_A - V_B) \propto x_A - x_B$

$(V_A - V_B) = K(x_A - x_B)$

when $x_A - x_B = 10$ We have $V_A - V_B = 10$

We get

10 = K10 $\Rightarrow$ K = 1

$\Rightarrow$ $V_A - V_B = (x_A - x_B)$ ....(1)

Now Let

$x_A - x_B = y$ ....(2)

On differentiating with respect to ‘t’ on both side.

$\Rightarrow$ $\frac{dx_A}{dt} - \frac{dx_B}{dt} = \frac{dy}{dt}$

$\Rightarrow$ Using (1) and (2)

$\frac{d(x_A - x_B)}{dt} = x_A - x_B$

$\frac{d(x_A - x_B)}{x_A - x_B} = dt$

$\Rightarrow$ $\left[\ell n(x_A - x_B)\right]_{10}^{20} = t$

t = (log$_e$2) sec ***Required Answer.***

13. (a)
Accelerations of particle and block are shown in figure.
Acceleration of particle with respect to block
= acceleration of particle – acceleration of block
$= (g \sin\theta\,\hat{i} + g\cos\theta\,\hat{j}) - (g\sin\theta\,\hat{i}) = g\cos\theta\,\hat{j}$
Now motion of particle with respect to block
will be a projectile as shown.
The only difference is, g will be replaced by g cos θ

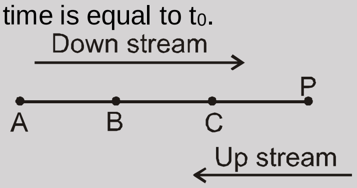
$\therefore$ PQ = Range (R) = $\dfrac{u^2 \sin 2\alpha}{g\cos\theta}$

PQ = $\dfrac{u^2 \sin 2\alpha}{g\cos\theta}$ **Ans.**

(b) Horizontal displacement of particle with respect to ground is zero. This implies that initial velocity of particle with respect to ground is only vertical, or there is no horizontal component of the absolute velocity of the particle.
Let v be the velocity of the block down the plane.
Velocity of particle with respect to block = $u\cos(\alpha+\theta)\,\hat{i} + u\sin(\alpha+\theta)\,\hat{j}$
Velocity of block $= -v\cos\theta\,\hat{i} - v\sin\theta\,\hat{j}$
$\therefore$ Velocity of particle with respect to ground = $\{u\cos(\alpha+\theta) - v\cos\theta\} + \{u\sin(\alpha+\theta) - v\sin\theta\}\,\hat{j}$
Now as we said earlier with that horizontal component of absolute velocity should be zero.
Therefore,
$u\cos(\alpha+\theta) - v\cos\theta = 0$
or $v = \dfrac{u\cos(\alpha+\theta)}{\cos\theta}$ (down the plane)

$v = \dfrac{u\cos(\alpha+\theta)}{\cos\theta}$ **Ans.**

14. At t = 0, raft (a float of timber) and motor boat are at point A. The velocity of raft is equal to velocity of stream.
At t = τ = 60 min, the motor boat is at point P and raft is at point B.
$\therefore$ The time taken by raft to reach from A to B = the time taken by motor boat to reach at P from A.
At $t = \tau + t_0$, both meet at point C,
So, the time taken by raft to reach at C from B is equal to the time taken by the motor boat to reach at C from P in upstream motion. This time is equal to t_0.



Let
v_A = actual velocity of motor boat,
v_B = actual velocity of stream = velocity of raft
$\therefore$ During down stream,
v_c = relative velocity of motor with respect to stream.
$\therefore$ $v_0 = v_A - v_B$
$\therefore$ $v_A = v_0 + v_B$
$\therefore$ $\tau = \dfrac{AP}{v_A} = \dfrac{AP}{v_0 + v_B}$
But AB = distance travelled by raft in time $\tau = v_B\tau$
During upstream,
$v_0 = v_A + v_B$
$\therefore$ $v_A = v_0 - v_B$
$\therefore$ PC = distance travelled by motor boat in upstream in time $t_0 = (v_0 - v_B)\,t_0$

BC = distance travelled by raft in time $t_0 = v_B t_0$
According to fig.

$\because$ AP – PC = AC = ℓ

or $(v_0 + v_B)\,\tau - (v_0 - v_B)\,t_0 = \ell$

or $v_0 t + v_B\, t - v_0 t_0 + v_B t_0 = \ell$ ....(i)

Also, $\because$ AB + BC = ℓ

or $v_B t + v_B t_0 = \ell$

or $v_B = \dfrac{\ell}{\tau + t_0}$ ....(ii)

From equation (i) and (ii) we get

$v_0 t + v_B t - v_0 t_0 + v_B t_0 = \ell$

or $v_0\tau + \dfrac{\ell}{\tau + t_0}\tau - v_0 t_0 + \dfrac{\ell}{t_0 + \tau} t_0 = \ell$

or $v_0\tau^2 + \tau\ell - v_0 t_0^2 + \ell t_0 = \ell\,(\tau + t_0)$

or $v_0\tau^2 - v_0 t_0^2 = \ell\,(\tau + t_0) - \tau\ell - t_0\ell$

or $\tau = t_0$

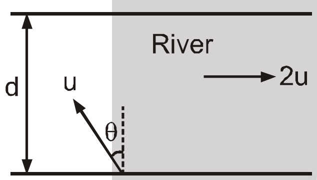
From (i) we have

$v_0\tau + v_B\tau - v_0 t_0 + v_B t_0 = \ell$

putting $\tau = t_0$

we get, $v_B = \dfrac{\ell}{2\tau}$.

15.



Let river velocity is 2u

time to cross river $t = \dfrac{d}{u\cos\theta}$

Drift $x = (2u - u\sin\theta)t = (2u - u\sin\theta)\dfrac{d}{u\cos\theta}$

Drift $x = (2\sec\theta - \tan\theta)d$

$\dfrac{dx}{d\theta} = (2\sec\theta\tan\theta - \sec^2\theta)d = 0 \quad \Rightarrow \quad 2\tan\theta = \sec\theta$

$\theta = 30^\circ$ with the river flow current

angle with stream $30^\circ + 90^\circ = 120^\circ$.